VOLUME LXXII NUMBER 5

FEB 9 1931

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
Yerkes Observatory of the University of Chicago

HENRY G. GALE
Ryerson Physical Laboratory of the University of Chicago

DECEMBER 1930

A STUDY OF STELLAR HYDROGEN LINES AND THEIR RELATION TO THE STARK EFFECT - - - - - - - - - - - - *C. T. Elvey and Otto Struve* 277

ON THE VARIATION OF THE INTENSITIES OF SPECTRAL LINES IN α^2 CANUM VENATICORUM - - - - - - - - - - - - - - *A. Markov* 301

MAGNITUDES OF SOUTHERN COMPARISON STARS FOR EROS *F. H. Seares, B. W. Sitterly, and Mary C. Joyner* 311

RECENT CHANGES IN THE ABSORPTION SPECTRUM OF 17 LEPORIS *Otto Struve* 343

REVIEWS

Theory of the Potential, W. D. MacMillan (Warren Weaver), 346.

INDEX - 349

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

December 1930 Vol. LXXII, No. 5

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
Yerkes Observatory of the University of Chicago

HENRY G. GALE
Ryerson Physical Laboratory of the University of Chicago

WITH THE COLLABORATION OF

WALTER S. ADAMS, Mount Wilson Observatory
JOSEPH S. AMES, Johns Hopkins University
ARISTARCH BELOPOLSKY, Observatoire de Pulkovo
WILLIAM W. CAMPBELL, Lick Observatory
HENRY CREW, Northwestern University
CHARLES FABRY, Université de Paris
ALFRED FOWLER, Imperial College, London
CHARLES S. HASTINGS, Yale University
HEINRICH KAYSER, Universität Bonn
ALBERT A. MICHELSON, University of Chicago
ROBERT A. MILLIKAN, Institute of Technology, Pasadena
HUGH F. NEWALL, Cambridge University
HENRY N. RUSSELL, Princeton University
FRANK SCHLESINGER, Yale Observatory
SIR ARTHUR SCHUSTER, Twyford
FREDERICK H. SEARES, Mount Wilson Observatory

The Astrophysical Journal is published by the University of Chicago at the University of Chicago Press, 5750 Ellis Avenue, Chicago, Illinois, during each month except February and August. ¶ The subscription price is $6.00 a year; the price of single copies is 75 cents. Orders for service of less than a half-year will be charged at the single-copy rate. ¶ Postage is prepaid by the publishers on all orders from the United States, Mexico, Cuba, Porto Rico, Panama Canal Zone, Republic of Panama, Dominican Republic, Canary Islands, El Salvador, Argentina, Bolivia, Brazil, Colombia, Chile, Costa Rica, Ecuador, Guatemala, Honduras, Nicaragua, Peru, Hayti, Uruguay, Paraguay, Hawaiian Islands, Philippine Islands, Guam, Samoan Islands, Balearic Islands, Spain, and Venezuela. ¶ Postage is charged extra as follows: for Canada and Newfoundland, 30 cents on annual subscriptions (total $6.30); on single copies, 3 cents (total 78 cents); for all other countries in the Postal Union, 50 cents on annual subscriptions (total $6.50), on single copies, 5 cents (total 80 cents). ¶ Patrons are requested to make all remittances payable to The University of Chicago Press, in postal or express money orders or bank drafts.

The following are authorized agents:

For the British Empire, except North America, India, and Australasia: The Cambridge University Press, Fetter Lane, London, E.C. 4. Yearly subscriptions, including postage, £1 9*s*. 3*d*. each; single copies, including postage, 3*s*. 9*d*. each.

For Japan: The Maruzen Company, Ltd., Tokyo.

For China: The Commercial Press, Ltd., Paoshan Road, Shanghai. Yearly subscriptions, $6.00; single copies, 75 cents, or their equivalents in Chinese money. Postage extra, on yearly subscriptions 50 cents, on single copies 5 cents.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when losses have been sustained in transit, and when the reserve stock will permit.

Business Correspondence should be addressed to The University of Chicago Press, Chicago, Illinois.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, Yerkes Observatory, Williams Bay, Wisconsin.

The cable address is "Observatory, Williamsbay, Wisconsin."

The articles in this Journal are indexed in the *International Index to Periodicals*, New York, N.Y.

Entered as second-class matter, January 17, 1895, at the Post-Office at Chicago, Ill., under the act of March 3, 1879.
Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 15, 1918.

PRINTED IN THE U.S.A.

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME LXXII DECEMBER 1930 NUMBER 5

A STUDY OF STELLAR HYDROGEN LINES AND THEIR RELATION TO THE STARK EFFECT

By C. T. ELVEY and O. STRUVE

ABSTRACT

We have examined the *wings of the hydrogen lines in early type* stars and have found them to be much *more extensive* than has heretofore been supposed. Spectrograms have been taken with Eastman Process plates to obtain high contrast, and with rather short exposures in order that the density of the continuous spectrum would fall within the range giving the largest deflections when the plate is analyzed with the microphotometer. Spectrograms taken with several instruments were compared in order to eliminate the selective absorptions in the glasses of the optical systems. A comparison of the spectrograms of α Lyrae, α Cygni, 67 Ophiuchi, and 85 ι Herculis shows that *in α Lyrae and perhaps in ι Herculis the red wing of Hε*, λ 3970, *overlaps the violet wing of Hδ*, λ 4102. The observations make *it doubtful that any of the continuous spectrum between Hδ and Hγ is free from the wings* of those lines. In α Cygni and in 67 Ophiuchi the overlapping is much less pronounced.

These very *extensive wings* of the hydrogen lines *indicate the influence of Stark effect.* This is shown by comparing the hydrogen lines of 85 ι Herculis (dwarf), which have very extensive wings, with those of 67 Ophiuchi (giant), which have narrow wings. The total absorptions of *Hδ* in these two stars differ greatly. This may in part be due, also, to Stark effect, but it is probable that another phenomenon is present which makes the total absorptions of the hydrogen lines larger in the dwarfs than in the giants.

Two pairs of stars, the first a giant and the second a dwarf in each case, *7 ε Aurigae −88γ Pegasi* and *50 α Cygni−24 γ Orionis*, selected according to total absorptions of the hydrogen lines and irrespective of the spectral type, show that the *contours differ greatly*. This is interpreted as being due to Stark effect.

The overlapping of the wings of the hydrogen lines, possibly as far as *Hγ*, adds *a complication to the determination of the temperatures and color indices* for the stars of classes A and B.

An attempt was made to *compute the influence of ionic fields upon the contour of a hydrogen line.* For the purpose of the computation it was assumed that the undisturbed line has approximately the shape of *Hγ* in *30η* Leonis. The variation of the *absorption coefficient in the line* was determined as *a function of optical depth*, on the assumption that the pressures are as given in the work of E. A. Milne. For simplicity the absorption coefficient of the continuous spectrum, the concentration of absorbing atoms, and the temperature were taken as constant. *The ratio of the absorption coefficients in the line*

and in the continuous spectrum was evaluated as a function of optical depth, and the contour of the line was computed by means of formulae developed by Sir Arthur Eddington. The result indicates that (1) *Stark effect* produces an appreciable *change in the contour:* the center of the line is slightly flattened and the wings are greatly widened; (2) the *total energy* absorbed in the line is materially *increased;* (3) the *flattening* of the line *in the center* is not proportional to the spread of the wings—this accounts for the existence of distinct lines up to the eighteenth member of the Balmer series in Vega; (4) the *spread of the wings should increase toward higher members* of the series, but not as rapidly as the simple formulae used previously had indicated; and (5) the wide wings observed by us in certain stars agree in extent with the theoretical contours.

A recent investigation of the contours of several members of the Balmer series of hydrogen in the spectrum of Vega, in which we found that the lines $H\beta$ to $H\eta$, inclusive, had nearly the same contours, prompted us to a more critical study of the hydrogen lines in stellar spectra. The results from the Balmer lines of Vega did not seem to conform to the demands of the Stark effect, and since the helium lines give unmistakable evidence of the presence of this effect in an average star, we felt that there must be another explanation. Since the Stark effect manifests itself in the wings of the lines and since the absorption line of a star is made up of the absorptions from different levels in the reversing layer, we should expect extensive wings to be produced by the atoms in the denser layers where the ionic fields are large. However, these wings would probably be relatively faint since the atoms in the upper levels will superimpose an absorption line practically free from Stark broadening.

Before discussing the observations of the extreme wings of the lines it will be best to take up the method used in determining the contour of an absorption line. The spectrogram is run through the microphotometer with such a speed that the photographic density is accurately registered on the tracing. A typical density-curve is that shown in Figure 1 for the $H\gamma$ line in the spectrum of 66 α^2 Geminorum. The next problem is that of drawing the continuous background across the absorption line. This, as was recently shown by Miss E. T. R. Williams,[1] is probably the cause of the systematic deviations of the total absorptions among the different observers. This is amply illustrated by the following: On the density-curve in Figure 1 two positions of the continuous background are chosen 1 mm apart, which is probably within the range that two observers might reconstruct the continuous spectrum. The usual procedure of

[1] *Astrophysical Journal,* **72,** 127, 1930.

reducing the contour was employed for each of the two positions of the background, and the results are shown in the contour in Figure 1 lower right-hand diagram. The reduction-curve, which is obtained by plotting the deflections for the standard intensities against their relative values expressed in magnitudes, is shown by the curve in the lower left-hand part of Figure 1. An illustration of the method is as follows: Take a point A on the contour, measure the deflection of the galvanometer, $AB = 21.0$ mm, and then the deflection for the

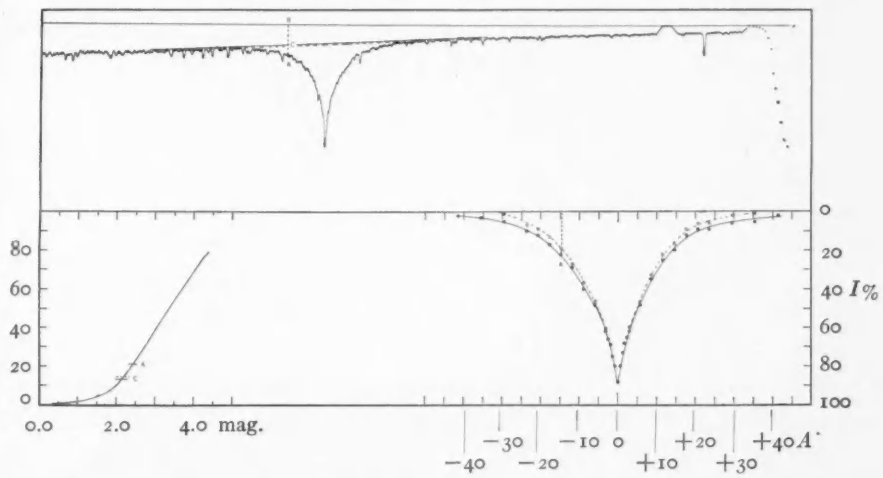


FIG. 1.—Examples of a microphotometric tracing (top), the reduction-curve (lower left-hand) obtained from the calibration at the right of the tracing, and the contours of the absorption line assuming the two different positions of the continuous background across the line in the tracing.

continuous background, $BC_1 = 13.5$ mm and $BC_2 = 14.5$ mm, corresponding to the two positions chosen. From the reduction-curve the stellar magnitudes are obtained for each measurement. The relative intensity of point A as compared with the continuous background is the difference of the stellar magnitudes, or 0.26 and 0.22 mag., respectively, for the two cases. This value of the amount of absorption, expressed in magnitudes, is readily changed to percentages of absorption by means of a curve plotted on semilog paper, the relationship on such a plot being represented by a straight line. The corresponding values of the intensity of the point A on the contour are 22 and 19 per cent of absorption of the continuous spectrum.

The resulting contours are shown in the lower right-hand corner of Figure 1, the continuous curve referring to the solid background and the dotted curve to the background shown by dashes. It is seen that there is a marked difference in the contours, the one showing much wider wings than the other. Such a displacement of the background has very little effect upon the center of the line; a difference of 0.04 mag. in I_m in the vicinity of $I_m = 2.0$ mag. means only a change of less than unity in $I_{\%}$, but when I_m is in the vicinity of 0.10 mag. the change is nearly four units. The effect upon the width of the line is a change from 60 A for the continuous contour to 47 A for the dotted one, at a position of 5 per cent of absorption. The large systematic differences among various observers for the total absorptions of the hydrogen lines are easily accounted for since the wings of the lines have a great influence on the total area of the contour. The total absorptions for the two contours given in Figure 1 are 18.0 and 15.3, expressed in Ångstroms of complete absorption of the continuous spectrum. The area of one contour is 18 per cent larger than that of the other. The value of 18.0 A, obtained by using a background as shown by the continuous line, corresponds closely to the value of 17.90 A by Elvey,[1] determined from the same spectrogram but from a different microphotometric curve.

Since the wings of the lines play such an important part in determining the total absorption, we have attempted to measure them with considerable accuracy. The hydrogen lines are usually deep, and in order to obtain the residual intensities of the centers of the lines it is necessary to overexpose the continuous spectrum. This seems to have been the case in all former investigations on this subject, and, in consequence, the outermost wings have usually been cut off too close to the central wave-lengths. It appears that much better results for the wings can be obtained by neglecting the centers of the lines altogether, and by adjusting the exposure time in such a way as to obtain maximum efficiency for measurements of the wings of the lines. The differences of intensity to be measured are quite small, and in order to make them as evident as possible we have obtained several spectrograms on Eastman Process plates with the single-prism Bruce spectrograph. The exposures were shorter

[1] *Ibid.*, **71**, 191, 1930.

than the normal, and the development was prolonged so as to increase the contrast. The spectra of two stars were obtained in this manner: 50 α Cygni, which is a supergiant of class A2 and has very narrow lines, and 3 α Lyrae, a typical star of class Ao having very wide hydrogen lines but sharp and faint metallic lines. As far as we can tell from our spectrograms of a dispersion of three prisms these two stars are practically free from axial rotation perpendicular to

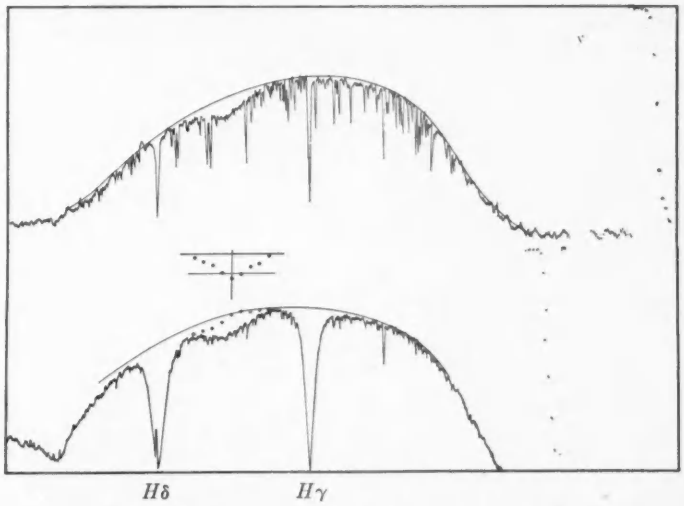


FIG. 2.—Microphotometric curves of spectra of α Cygni (top) and α Lyrae. Between the two curves is the contour of the selective absorption band of the glass of the optical system. The curve for α Lyrae has been corrected for this band.

the line of sight. The microphotometric tracings of the two spectrograms of these stars are shown in Figure 2. One at once notices the marked differences between the two spectra, especially in the extent of the wings of the hydrogen lines. On this scale the hydrogen lines in α Cygni appear to have no wings, while in α Lyrae they are very extensive. Before it is possible to determine the extent of the wings the effect of the troublesome absorption band near λ 4200, owing to the optical glasses of the telescopic and spectrographic systems, must be eliminated. The continuous background of α Cygni can be drawn with considerable precision, and from the tracing we obtain the contour of the absorption band. Its value is taken here from the

Process plates since the values given by Elvey and Zug[1] were obtained with the use of Eastman-40 plates. The contour of this band is shown between the two microphotometric tracings in Figure 2. The central absorption amounts to 0.13 mag. With the use of the reduction-curve for the tracing of the spectrum of α Lyrae we are able to reconstruct the background, freed from the selective absorption of the glass. This is shown by the dots on the tracing. We have put in a continuous background for the spectrum of α Lyrae, using as a guide the general trend of the background for the spectrum of α Cygni. The background, as we have drawn it, is tangent to the curve at one point between $H\gamma$ and $H\delta$, which indicates that the wings of the lines extend nearly to each other, and perhaps even overlap. It would be possible to draw the background in such a way that it would touch more of the continuous spectrum between $H\gamma$ and the point of contact shown, but this would make it necessary to place the maximum of intensity of the continuous spectrum farther toward the violet, and would also cause the contour of $H\gamma$ to be asymmetrical. As it is, we have placed the maximum for α Lyrae considerably to the violet of that for α Cygni. Such a shift is justified since the temperature of α Cygni is thought to be about 3000° lower than that of α Lyrae. Atmospheric scattering can be neglected since the spectrograms of both stars were obtained under about the same conditions (the same night and both stars at high altitudes).

The contour of $H\gamma$ for α Lyrae has been reduced from the microphotometric tracing, and is shown in Figure 3. The wings of the line were obtained from spectrograms made on Eastman Process plates and are shown by the dots. The core of the line is obtained from Eastman-40 plates since the magnitude range of this emulsion is greater than that of the Process plate. As was shown before, small errors in the placing of the continuous background, when it is rather dense on the spectrogram, make very little difference in the intensities of the parts of the line where the absorption is large. These measures are shown by circles; and it will be noticed that the points join satisfactorily and show that it is possible to determine the contour of a line by the use of different plates for the core and for the wings of the line.

[1] *Ibid.*, **70**, 243, 1929.

The contour of $H\gamma$ in α Lyrae, as reproduced, shows that the wings of the line extend to 80 A on either side of the center at an intensity of 1 per cent of absorption of the continuous spectrum. This is about twice the extent heretofore recorded for this line. The curve drawn through the points is a simple exponential curve of the form $I = I_0\, e^{-k(\lambda-\lambda_0)}$, where $I_0 = 85$ and $k = 0.077$.

In order to have a spectrum free from the selective absorption of glass we used the Zeiss doublet of six-inch aperture, made of UV

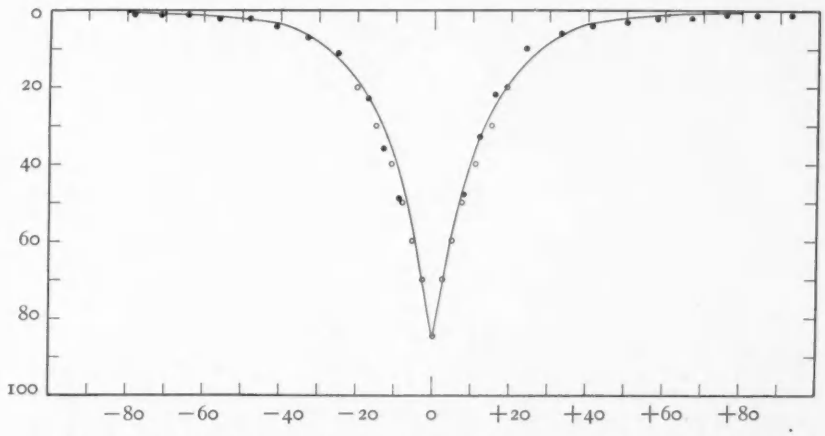


FIG. 3.—Contour of $H\gamma$ in α Lyrae. The dots were obtained from a spectrogram on Eastman Process plate and the circles from an Eastman-40 plate. The ordinates are percentages of absorption and the abscissae are Ångstrom units. The curve is of the form $I = I_0\, e^{-k(\lambda-\lambda_0)}$, where $I_0 = 85$ and $k = 0.077$.

glass and equipped with a 30° and a 15° prism (combined to give 45°) of UV glass, to obtain objective-prism spectra of α Cygni and α Lyrae. The microphotometric tracings of these two spectrograms are shown in Figure 4. It is seen from the tracing of the spectrum of α Cygni that the absorption band at λ 4200 due to glass is absent. However, there is a band farther to the violet, beginning at about λ 3900 and extending to $H\lambda$, λ 3734. This is perhaps the cyanogen band with the head at λ 3885, but there may, also, be present some absorption in the glass.

In the spectrum of α Cygni there is very little evidence of overlapping of the wings of the hydrogen lines until the higher members of the series are reached. On the other hand, in the spectrum of α

Lyrae the overlapping is definite for the lines to the violet of $H\delta$, and it is uncertain that any of the spectrum between $H\gamma$ and $H\delta$ is free from the wings of those two lines.

In determining the relative temperatures of two stars such as α Cygni and α Lyrae from the distribution of the energy of the continuous spectrum we see that if points to the violet of $H\gamma$ are compared, a considerable uncertainty enters, owing to the fact that in the case of α Lyrae the original background of the continuous spec-

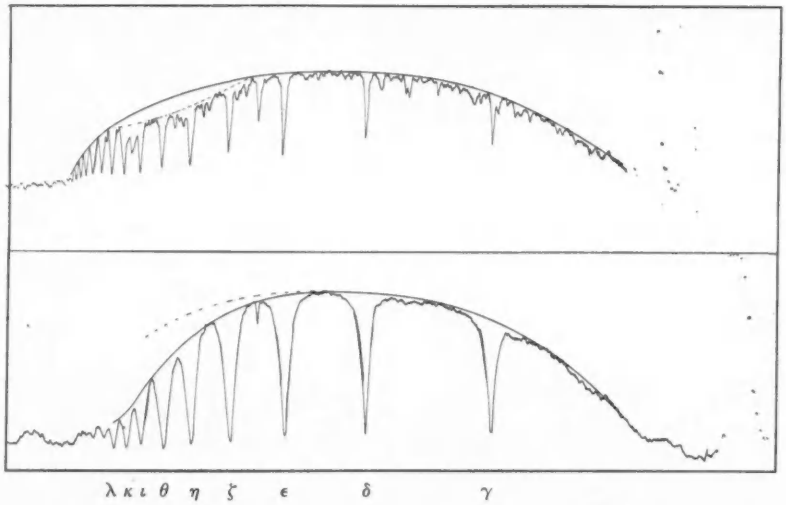


FIG. 4.—Microphotometric tracings of objective prism (45°) spectra with a UV camera of α Cygni (top) and α Lyrae. In α Cygni the lines are narrow and there is no overlapping of the wings of the hydrogen lines till the higher members of the series are reached. In α Lyrae there is a confluence of the wings of the lines to the violet of $H\delta$.

trum is not measured. Thus, if the temperature of α Cygni is taken as known, the temperature of α Lyrae is found to be too low, and if α Lyrae is taken as the standard, then the temperature of α Cygni is too high. Likewise, the color indices of the class-A stars with extensive hydrogen lines will be larger than for those with narrow lines.

For further tests of the extensive wings of the hydrogen lines we were fortunate in having two spectrograms taken by Struve with the Coudé spectrograph of UV glass in connection with the 100-inch telescope, for a study of the Stark effect in the helium lines. The two stars whose spectra were used are 67 Ophiuchi, a giant of spectral type B5, and 85 ι Herculis, a dwarf of class B3. Microphotometric

tracings of these two spectra are shown in Figure 5. In the spectrum of ι Herculis the Stark effect is very pronounced, as is shown by the fact that the helium lines of the diffuse series are wide and diffuse while the lines of the principal series are sharp. In 67 Ophiuchi, on the other hand, the lines of both series are sharp, which indicates that Stark effect is almost absent. It will be noticed in referring to Fig. 5 that the hydrogen lines of 67 Ophiuchi are much narrower than those of ι Herculis. The wings of the lines of the latter star are so wide that it seems almost certain that those of $H\delta$ and $H\epsilon$ overlap.

The contours of $H\delta$ in these two stars (Fig. 6) differ primarily in total absorption, the giant having a much weaker line than the

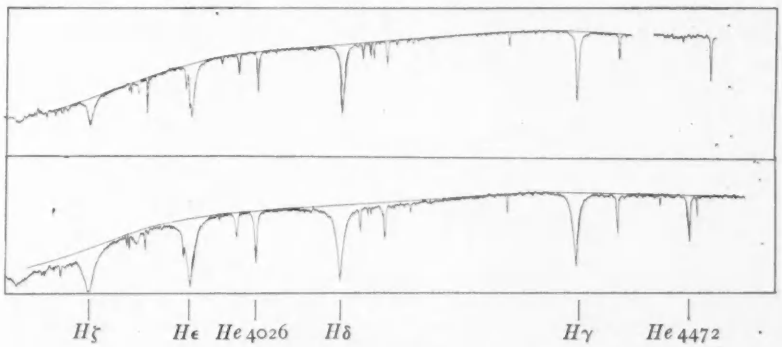


FIG. 5.—Microphotometric tracings of spectra of 67 Ophiuchi (top) and of 85 ι Herculis made with the Coudé spectrograph of UV glass of the Mount Wilson Observatory.

dwarf. The wings of the hydrogen lines in ι Herculis are much wider than those for 67 Ophiuchi, as the Stark effect demands, but the difference in total absorption makes it difficult to compare the two lines. The consequences will be discussed in the theoretical part of this paper.

The contours were plotted on semilog paper (Fig. 7), where the ordinates are ratios of I/I_0 and the abscissae are Ångstrom units. The contours of the absorption lines, especially of hydrogen, have been observed to have the form of a simple exponential curve, which is a straight line when plotted as above. The plot in Figure 7 shows that the central parts of the line fall on an exponential curve of the type $I = I_0\, e^{-k(\lambda-\lambda_0)}$; but toward the wing of the line there is a break showing that the contour follows some other law.

The difficulty encountered above in the case of ι Herculis and 67

Ophiuchi, where the total absorptions were so different as to make the lines practically incomparable, suggested that a more definite result might be obtained by comparing lines with approximately

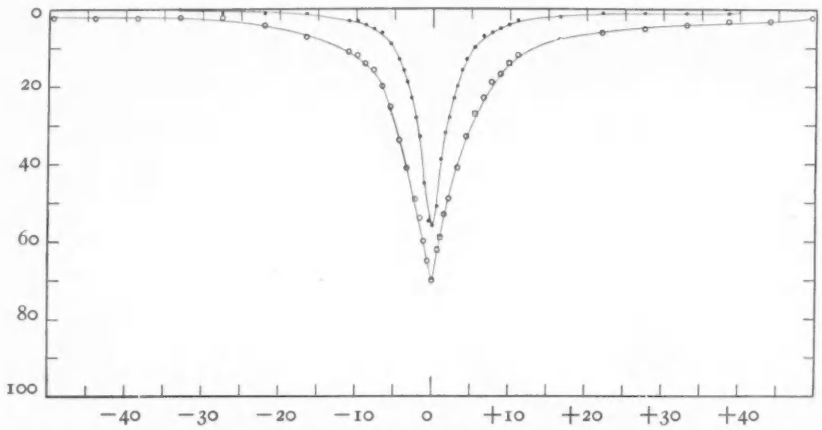


FIG. 6.—Contours of $H\delta$ in 67 Ophiuchi (dots) and in 85 ι Herculis. The ordinates are percentages of absorption and the abscissae are Ångstrom units.

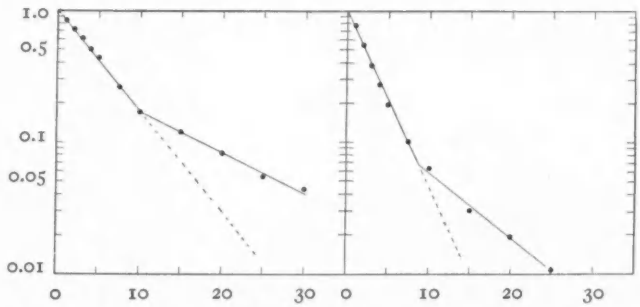


FIG. 7.—The data of the contours in Fig. 6 are plotted on semi-log paper, the ordinates are ratios of I/I_0, and the abscissae are Ångstrom units. Iota Herculis is on the left and 67 Ophiuchi is on the right.

equal total intensities. If the number of atoms present is the sole agent responsible for the widening of the line, then two contours with equal total intensities should be identical, irrespective of the absolute magnitudes of the stars. On the other hand, Stark effect demands a definite change in the shape of the contour, depending upon the atmospheric density or the absolute magnitude.

We have shown in a former investigation[1] that the $H\beta$ line in the spectrum of 7 ϵ Aurigae has a contour which approaches very closely to the theoretical contour of an absorption line as given by A. Unsöld.[2] Most of the hydrogen lines in stellar spectra do not have a contour similar to the theoretical one. We have chosen for comparison the contour of $H\beta$ in 88 γ Pegasi. This line has about the same

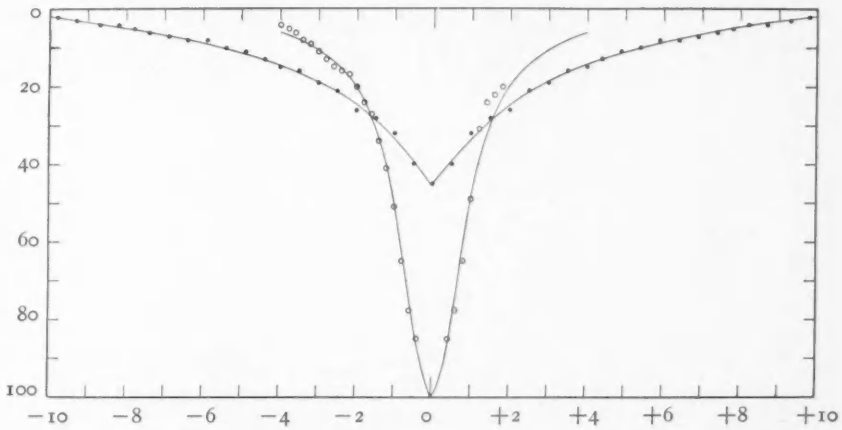


FIG. 8.—Contours of $H\beta$ in the spectrum of 7 ϵ Aurigae (circles) and in 88 γ Pegasi (dots). The curve for 7 ϵ Aurigae is the Unsöld contour showing that there is no broadening of the line due to axial rotation or to Stark effect. The contour for 88 γ Pegasi shows a broadening which is interpreted as that of Stark effect; rotational broadening is absent, as is shown by other lines in the spectrum which are not appreciably influenced by electric fields.

total absorption as the one in 7 ϵ Aurigae (Fig. 8). The star 7 ϵ Aurigae is known to be a supergiant, and the density of its atmosphere is very low; hence the chance of an absorbing atom being in the neighborhood of a charged particle is very small, that is, there is little Stark effect. On the other hand, 88 γ Pegasi is known to show a large Stark effect[3] in the helium lines, and it is quite reasonable to suppose that the ionic fields should show their effect on the hydrogen lines. The difference in the shapes of the two contours (Fig. 8) is obvious, and it agrees very closely with that demanded by the theory of the Stark effect, as is illustrated in Figure 10.

[1] O. Struve and C. T. Elvey, *Nature*, **125**, 308, 1930; *Astrophysical Journal*, **71**, 136 1930.

[2] *Zeitschrift für Physik*, **44**, 793, 1927; **46**, 768, 1928.

[3] Struve, *Astrophysical Journal*, **70**, 85, 1929; *ibid.*, p. 237, 1929.

To give another example we have taken the contour of $H\gamma$ in 50 α Cygni, a supergiant of spectral class A2, and compared it with the same line in a star of class B, 24 γ Orionis, which shows the Stark effect. The contours are given in Figure 9 and were determined, as were those of Figure 8, with the aid of Eastman Process plates to obtain the faint wings of the lines.

It seems to us that the evidence is conclusive that the contour of a line affected by Stark broadening is very different from one which

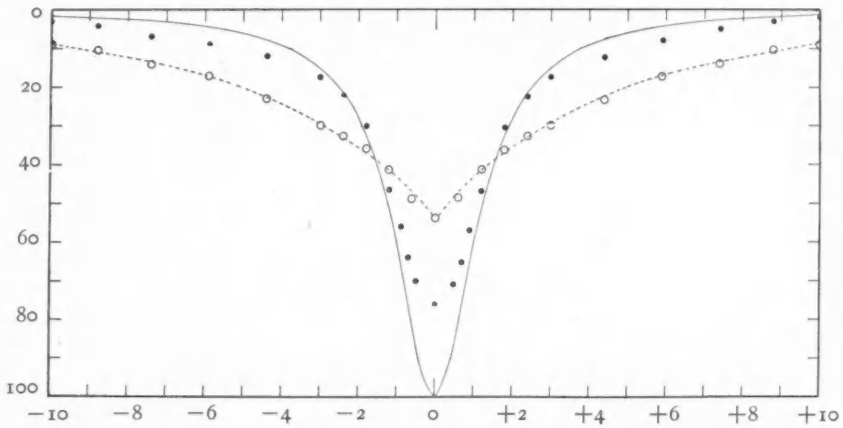


FIG. 9.—Contours of $H\gamma$ in α Cygni (dots) and in γ Orionis (circles). The continuous curve is the Unsöld contour for $NHf=5\times10^{17}$. The line in γ Orionis shows a broadening which is due to the Stark effect.

originates in an atmosphere where the charged particles, or ions, per unit volume are few.

THEORETICAL CONSIDERATIONS

It is a well-established fact that the contours of the hydrogen lines in stars cannot be reproduced by the Schuster-Schwarzschild approximation in which is used the absorption coefficient applicable to gases with "radiation-damping."[1] Speaking broadly, the observed lines are usually shallower in the center and wider in the wings than the "Unsöld contours." This departure from theory was explained by A. Unsöld[2] and by Struve[3] as being due to Stark effect produced

[1] Unsöld, *Zeitschrift für Physik*, **59**, 353, 1930; Elvey, *loc. cit.*

[2] *Op. cit.*, **59**, 353, 1930.

[3] *Astrophysical Journal*, **69**, 173, 1929.

by perturbations in the radiating atoms caused by the fields of neighboring ions. The behaviors of the helium lines in stellar spectra have given us considerable confidence in the existence of an appreciable ionic Stark effect in the reversing layer of early type stars. It is therefore, a priori, probable that the hydrogen lines will also be found to be subject to Stark effect.

Several investigators have attempted to study the Stark effect by means of the hydrogen lines. Unsöld[1] has shown that in the solar spectrum the Balmer lines give contours in agreement with the demands of the Stark effect. The stellar data has been scrutinized by Elvey[2] and by Miss E. T. R. Williams,[3] and their results appeared to confirm the existence of a large Stark effect. On the other hand, a recent investigation by Miss C. J. Anger[4] has been only partially favorable to this hypothesis. It is therefore necessary to investigate the theory of the hydrogen lines in greater detail than has been done heretofore.

We may start by summarizing the observational data:

1. Contrary to expectation, the central intensity of the Balmer lines does not decline from $H\alpha$ toward the higher members of the series. In the majority of stars of early type the intensity changes very little from $H\alpha$ to $H\epsilon$ and then gradually decreases toward the limit of the series.

2. The great number of Balmer lines in absorption observed in stellar spectra has been regarded as fatal to the hypothesis of an ionic Stark effect. Over thirty hydrogen lines were observed in ζ Tauri by R. H. Curtiss.[5] In α Lyrae we have observed eighteen members. Even in stars known from the appearance of the helium lines to show the Stark effect there is a large number of hydrogen lines in absorption.

3. The shapes of the contours of the hydrogen lines differ appreciably from the Unsöld curve. Elvey found empirically that a simple

[1] *Op cit.*, **59**, 353, 1930.

[2] *Loc. cit.*

[3] *Harvard College Observatory Circular*, No. 348, 1929.

[4] Paper presented at Chicago meeting of the American Astronomical Society, September, 1930; *Harvard College Observatory Circular*, No. 352, 1930.

[5] *Publications of the Astronomical Observatory of the University of Michigan*, **3**, 256, 1923.

exponential function, $I = I_0\, e^{-k(\lambda-\lambda_0)}$, fits the observations satisfactorily. In the preceding section of this paper it has been shown that dwarfs have flatter contours and more extended wings than giants.

4. The wings are very pronounced in the dwarfs and nearly absent in the giants. In α Lyrae the wings of $H\epsilon$ and of $H\delta$ overlap and those of $H\gamma$ approach dangerously close to the wings of $H\delta$.

5. The total amount of energy absorbed is a function of absolute magnitude for any given spectral subdivision,[1] and of temperature for any given absolute magnitude. Miss Anger has suggested that the phenomenon associated with absolute magnitude may be due to an increase in the total absorption produced by Stark effect.

6. The total absorption has a tendency to increase from $H\alpha$ to $H\epsilon$, decreasing from there on toward the limit of the series.[2]

7. The width of the lines was formerly believed to change little, if at all, along the series. We now know that this refers to the core of the line alone. The wings overlap for most stars with strong hydrogen lines beginning about $H\delta$ (supergiants are excepted), and there is no information available concerning their extent.

It is evident that the simplified treatment of the Stark effect for a gas at uniform pressure[3] does not account for all of the observed facts. The existence of eighteen members of the Balmer series in a star which would naturally be expected to show Stark effect is particularly startling. In the laboratory the higher members of the series in emission blend together at comparatively low pressures and form a continuous band even when the lower members are not greatly broadened. By analogy we might be led to expect that the absorption lines of a star would behave similarly. The discrepancy obviously arises from the fact that in the laboratory we are dealing with a gas at constant pressure and temperature, while in the reversing layer of a star the pressure varies from something like 10^{-11} atmospheres at the top, where the chromosphere sets in, to a value of 10^{-2}

[1] Payne and Williams, *Monthly Notices of the Royal Astronomical Society*, **89**, 526, 1929; Elvey, *op. cit.*, **71**, 205, 1930; Anger, *loc. cit;* see also a dissertation by Yngve Öhman, *Meddelanden från Astronomiska Observatorium Upsala*, No. 48, 1930.

[2] This may, however, be different, if true total absorptions could be measured for the higher members.

[3] *Astrophysical Journal*, **69**, 173, 1929.

atmospheres, where the opacity of the material is great enough to cut off the light of the underlying sections.

Our problem consists of determining the theoretical contour of a line subject to Stark effect in an atmosphere in which the pressure increases with depth. This involves the application of an absorption coefficient that varies with depth. We shall adopt the method of Sir Arthur Eddington[1] with all the reservations and assumptions inherent in this type of problem, as stated by him in the first part of his paper.[2]

Let us define by l (a function of λ) the mass-absorption coefficient within the line; by κ, the mass-absorption coefficient of the continuous spectrum applicable both inside and immediately outside the line. We shall assume that κ is independent of λ, which is justified since we are concerned with a comparatively narrow range of λ.

Following Eddington, we designate

$$\eta = \frac{l}{\kappa} \tag{1}$$

and

$$\tau = -\int_{\circ}^{r} \kappa \rho dr \,, \tag{2}$$

so that τ is the optical depth for light just outside the absorption line. If the absorption coefficients in the line and outside of it are given per unit of length, we have

$$\sigma = l\rho \,,$$
$$\psi = \kappa\rho \,,$$

where ρ is the total density. Consequently

$$\eta = \frac{l}{\kappa} = \frac{\sigma}{\psi} \,. \tag{3}$$

[1] *Monthly Notices of the Royal Astronomical Society*, **89**, 620, 1929.

[2] In other words, we adopt Eddington's "very special case in which the number of atoms in an excited state is conditioned wholly or mainly by the line that is being studied" (*ibid.*, p. 634). This limitation is unfortunate but cannot be avoided. There are, however, reasons for believing that actual lines do form as described above.

But the absorption coefficient in the line per unit of length is

$$\sigma = C_1\varphi(\lambda-\lambda_0)N' ,$$

where N' is the number of absorbing atoms. Similarly,

$$\psi = C_2 N ,$$

where N is the number of all atoms and C_2 may itself be a function of N. Consequently

$$\eta = \frac{C_1\varphi(\lambda)}{C_2}\frac{N'}{N} , \tag{4}$$

and is a function of the concentration of absorbing atoms N'/N. This ratio depends upon the conditions of ionization in the star. The concentration has a maximum somewhere within the reversing layer, but for simplicity we shall assume that N'/N is constant throughout the atmosphere. This simplifies the computations greatly, since otherwise it would be necessary to use Milne's ionization formulae in the expression for η.

Our next step is to fix the absorption coefficient of the continuous spectrum, κ or ψ. Milne[1] has investigated it, and has shown that in all probability

$$\kappa \sim \frac{p'}{T^{9/2}} , \tag{5}$$

where p' denotes the electronic pressure at any level. However, the simple assumption

$$\kappa = \text{Const} = \bar{\kappa} \tag{6}$$

has given fair results in his discussion of stars of type A, and while there is considerable evidence in favor of equation (5), we shall adopt $\kappa = \text{Const}$ since it tends to simplify the computations. Consequently

$$\eta \sim l \sim \sigma ,$$

and it is the variation, with optical depth, of the absorption coefficient in the line that we wish to investigate.

[1] *Monthly Notices of the Royal Astronomical Society*, p. 17, 1928.

Since by assumption $\kappa = \bar{\kappa}$, we may directly apply Milne's expression for the optical depth as a function of the pressure,

$$\tau = \frac{\bar{\kappa}}{g} 2p' = \frac{\bar{\kappa}}{g} P \; ; \tag{7}$$

here g is the surface gravity (the variation of g with T is, of course, neglected), p' is the electronic pressure, and P is the total pressure.

We shall now specify the line under consideration. Suppose that we are dealing with $H\gamma$ in Sirius. Then $g = 1.04 \times 10^4$ cm/sec.^{2}; $T = 11{,}000°$. Milne gives for $\tau = 1/3$: $p_0' = 2.9 \times 10^3$ dynes/cm^2; $\bar{\kappa} =$ 0.61 cm^2/gr. We compute P from (7) as a function of τ (Table I).

TABLE I

τ	P dynes/cm^2	F	$\Delta\lambda_0$
0.0.....	0.00	0 e.s.u.	0.0 A
.05.....	0.08×10^4	27	1.6
.1.....	.17	37	2.2
.2.....	.34	58	3.5
.3.....	.51	76	4.6
.4.....	.68	93	5.6
.5.....	0.85	107	6.4
.6.....	1.02	121	7.2
.7.....	1.19	134	8.0
.8.....	1.36	146	8.8
0.9.....	1.53	158	9.5
1.0.....	1.70×10^4	169	10.2

To each pressure there corresponds a certain definite value of the average field strength,[1]

$$F = \left(\frac{p'(\text{atm.})}{3.5 \times 10^{-10} . T} \right)^{2/3} = \left(\frac{P(\text{dynes})}{7.0 \times 10^{-4} . T} \right)^{2/3} = (0.13P)^{2/3} \; . \tag{8}$$

The results of the computation are shown in column 3 of Table I.

A field, F, splits a line into components, the distance from the normal wave-length being given by

$$\Delta\lambda = \frac{3h\lambda_0^2 F}{8\pi^2 mEc} [n(n_2 - n_1) - N(N_2 - N_1)] = \frac{9.2 \times 10^{-12} \lambda_0^2 F}{E} M \; . \tag{9}$$

[1] *Astrophysical Journal*, **70**, 90, 1929.

In this case $E=e$. For the expression, M, depending upon the quantum numbers, we use average values as computed by M. Vasnecoff.[1] In other words, we make the assumption that the line splits into two components[2] symmetrically spaced with respect to λ_0.

Line	M
$H\alpha$	4
$H\beta$	10
$H\gamma$	18
$H\delta$	28

From (9) we find that for $H\gamma$ the split in a field of 1 e.s.u. (at 1 cm distance) is $\Delta\lambda=0.06$ A (on both sides of λ_0). Since $\Delta\lambda \sim F$, we compute the values of $\Delta\lambda$ for the various fields given in Table I. The average splitting of $H\gamma$ due to the Stark effect is zero at the top of the reversing layer and reaches a value of more than 10 A at an optical depth of 1.0.

Within each layer of optical thickness $\Delta\tau$ we have all possible field strengths present. An infinitely small field has a probability of zero, since to each radiating atom there is at least one charged particle at finite distance, producing $F>0$. Similarly, the probability is zero for an infinitely large field. The frequency-curve of F, therefore, approaches zero asymptotically at $F=0$ and at $F=\infty$. Between these limits the frequency is greater than zero, reaching a maximum somewhere for a finite value of F. With Unsöld[3] we shall assume that the probability of a given value of F, corresponding to a splitting between $\Delta\lambda$ and $\Delta\lambda+d\Delta\lambda$, is

$$\Pi = e^{-\left(\frac{\Delta\lambda_0}{\Delta\lambda}\right)^{3/2}} d\left(\frac{\Delta\lambda_0}{\Delta\lambda}\right)^{3/2}.$$

[1] *Vestnik Kral. Ces. Spol. Nauk*, **2**, 1927.

[2] The real patterns of the Stark effect are, of course, much more complicated. Thus $H\gamma$ has a fairly strong central component which remains undisplaced in electric fields. Perhaps it would have been better to consider in our computations a line like $H\delta$ which has no appreciable central component. One might expect to find a difference in the contours of hydrogen lines having central components and hydrogen lines without central components. This should be tested in the laboratory.

[3] *Op. cit.*, **59**, 366, 1930.

The absorption coefficient in the line at a distance of $\Delta\lambda$ from λ_0 is proportional to this quantity divided by $d\Delta\lambda$:

$$\left.\begin{aligned}\sigma &= Ae^{-\left(\frac{\Delta\lambda_0}{\Delta\lambda}\right)^{3/2}} d\left(\frac{\Delta\lambda_0}{\Delta\lambda}\right)^{3/2} \frac{1}{d\Delta\lambda}, \\ &= A\,\frac{\Delta\lambda_0^{3/2}}{\Delta\lambda^{5/2}}\, e^{-\left(\frac{\Delta\lambda_0}{\Delta\lambda}\right)^{3/2}} = C_1 N' \varphi(\lambda-\lambda_0)\,.\end{aligned}\right\} \tag{10}$$

The constant A, which is independent of $\Delta\lambda$, will be determined from actual observations.

Let us consider the physical meaning of σ. The atoms in the reversing layer absorb quanta of energy $h\nu$. As long as ν does not differ much from ν_0, the quantum of energy is practically constant all along the contour of the line. Introducing Einstein's coefficient of probability for absorption, B_{ij}, we have[1]

$$\int \sigma d\nu = \frac{B_{ij} h\nu N_i \left(1-e^{-\frac{h\nu}{kT}}\right)}{C} = \text{Const.} \tag{11}$$

According to this, the integral of the absorption coefficient is independent of the form of the function φ in σ. In the reversing layer the pressure varies with optical depth, consequently the form of φ varies also, but the value of $\int \sigma d\lambda$ remains constant.

We have made the assumption that $\kappa = \text{Const}$, and consequently $\eta (= [l/\kappa] = [\sigma/\psi])$ depends upon l only, and varies because of Stark effect, as a function of τ. At the top of the reversing layer $P = 0$ and Stark effect is absent. In a supergiant star the pressures are low throughout the reversing layer, and approximately $\eta = \text{Const}$ for all values of τ. If we find a star which we believe to be very luminous, and consequently little disturbed by Stark effect, we shall have a reversing layer with a constant value of η, which value will apply to $\tau = 0$ of all other stars. Given such a contour for a very luminous star, we can directly apply the formulae of Eddington[2] and derive $\eta_{\tau=0}$ from

$$\frac{I_\nu}{I_0} = \frac{1+1.15\sqrt{1+\eta}}{1+\eta+1.15\sqrt{1+\eta}}\,. \tag{12}$$

[1] R. C. Tolman, *Statistical Mechanics*, p. 175, 1927.

[2] *Monthly Notices of the Royal Astronomical Society*, **89**, 624, 1929.

This formula neglects the fraction of the radiation which is not re-emitted by the absorbing atoms, but is lost by superelastic collisions.

We have chosen as a standard a contour similar to that of $H\gamma$ in the star η Leonis. This line resembles the theoretical contour of Unsöld,[1] and we have slightly modified it by making the central absorption a little greater. We shall assume that a supergiant star of class Ao has a contour as given in Table II. Column 3 contains the values of η derived from formula (12). They apply to all optical

TABLE II

$\lambda-\lambda_0$	I_ν/I_c	η
0 A	0.10	126
1	.20	30
2	.40	6
3	.60	2
4	.75	1.0
5	.80	0.6
7	.87	.35
10	.92	.20
15	.94	.15
20	.96	.10
30	.98	.05
40	0.99	.03
50	1.00	0.00

depths of our supergiant, but only to $\tau=0$ for a star of average luminosity. Since $\psi=\text{Const}$ and $\int\sigma d\lambda=\text{Const}$, we have also

$$\int\eta d\lambda=\text{Const.}$$

In an average star where the pressures increase with τ, as shown in Table I, Stark effect modifies $\eta=(\sigma/\psi)$ as shown by (10). We have

$$\int\eta d\lambda=\frac{1}{\psi}\int\sigma d\lambda\ .$$

We substitute σ from (10). The right-hand side, after integration, is $\frac{2}{3}(A/\psi)$. The expression on the left is obtained by a graphical integration of the values of η given in Table II. The constant A/ψ is thus determined. Substituting it into $\eta=(\sigma/\psi)$, where the functional relationship of σ to $\Delta\lambda$ is given by (10), we compute the data of Table III.

The table indicates in a crude way how η changes with τ. In

[1] *Zeitschrift für Physik*, **44**, 793, 1927.

reality the change is probably more gradual. Thus in the center of the line the table shows a sudden drop from 126 to 0. In reality the undisturbed value of η is gradually replaced by that effective for Stark broadening.

Our next step is the determination of the contour of the line. Eddington has given formulae for the determination of the residual intensity I_ν/I_0 if η varies according to the law

$$\frac{1}{1+\eta}=\alpha+\beta\tau\,. \tag{13}$$

TABLE III

$\lambda-\lambda$	η			
	$\tau=0$	$\tau=0.4$	$\tau=0.7$	$\tau=1.0$
0	126	0.0	0.0	0.0
1	30	0.2	.0	.0
2	6	1.5	0.9	.1
3	2	10.2	3.5	0.5
4	1.0	11.7	6.7	2.7
5	0.6	10.6	8.4	5.3
7	.35	8.3	8.2	6.5
10	.20	4.8	5.4	6.0
15	.15	2.5	2.7	3.9
20	.10	1.1	1.6	1.9
30	.05	0.3	0.6	0.9
40	.03	.1	.3	.5
50	.00	.1	.2	.2
60	.00	.0	.1	.1
70	.00	.0	.0	.1
80	0.00	0.0	0.0	0.0

This law does not coincide with the actual variation of η with τ, and it is hoped that Eddington's formulae may be extended to cover a type of variation, $\eta=F(\tau)$, similar to that shown by Table III. In the meantime we must be satisfied with a very rough approximation. Let us arbitrarily suppose that between two fixed values of τ (i.e., $\tau_1=0$ and $\tau_2=0.7$) η varies according to (13). Then α and β are determined for each value of $\Delta\lambda$ from the real η's of Table III; and the corresponding point on the contour is found from Eddington's formula,

$$\frac{I_\nu}{I_0}=\frac{\alpha-\sqrt{\tfrac{1}{3}\alpha}(2+\beta)\left(\dfrac{K_0'}{K_0}\right)}{1-\sqrt{\tfrac{1}{3}\alpha}\cdot 2\left(\dfrac{K_0'}{K_0}\right)}\,, \tag{14}$$

where K_0 and K_1 ($=-K_0'$) are Bessel functions, the argument being $x=\sqrt{12}\,a/\beta^2$. We have computed the contour for two cases, viz.: (1) $\tau_1=0$, $\tau_2=0.7$; and (2) $\tau_1=0$, $\tau_2=1.0$. The results are summarized below, and the first case is shown graphically in Figure 10.

TABLE IV

$\lambda-\lambda_0$	I_ν/I_0		
	No Stark Effect	With Stark Effect	
		$\tau_2=0.7$	$\tau_2=1.0$
0 A	0.10	0.36	0.28
1	.20	.48	.55
3	.60	.57	.....
3.5	.68	.....	.65
5	.80	.63	.....
10	.92	.66	.75
20	0.96	0.75	0.81

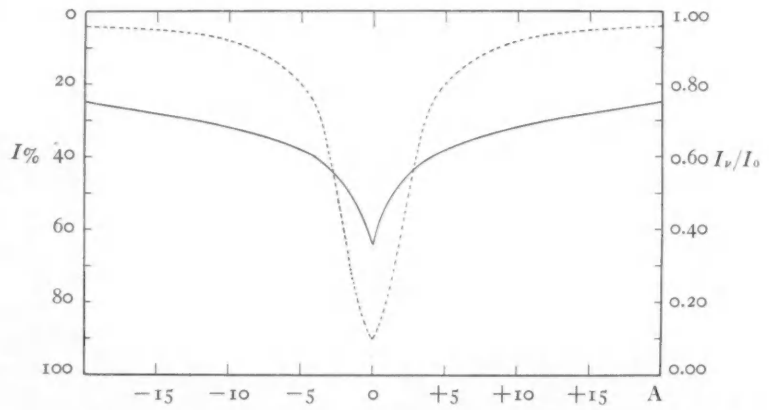


FIG. 10.—Changes in the contour of an absorption line of hydrogen due to the Stark effect. The assumed contour (dotted) is similar to $H\gamma$ in 30 η Leonis, a supergiant. Normal pressures in a stellar atmosphere produce Stark effect which changes the contour to that shown by the continuous line.

It should be noted that near the center of the line ($\lambda-\lambda_0<2.8$ A for $\tau=0.7$) we have $\beta>0$, and this is the case investigated by Eddington. Although for large values of τ the value of η from (13) becomes negative, Eddington has shown that this does not materially affect the results. For $\lambda-\lambda_0>2.8$ A we have $\beta<0$. In that case η in

formula (13) becomes discontinuous at $\tau=(\alpha/\beta)$ and it is not at once apparent that equation (14) is correct. However, at $\lambda-\lambda_0=2.8$ A, where $\beta=0$, we know the slope of the contour and the values computed from (14) for $\lambda-\lambda_0>2.8$ A agree with the extrapolated curve. For large values of $\lambda-\lambda_0$ we may obtain a check by another method. Since η varies but slowly in the wings, we may assume that in the surface layers of the atmosphere $\eta=0$, but beginning from a certain value of τ, $\eta=$Const. Applying Eddington's formulae for a star where the absorbing material does not extend to the surface,[1] we obtain the following results, under two assumptions concerning the critical values of τ: At the point $\lambda-\lambda_0=20$ A, $\eta=0$ for $0<\tau<0.2$ and 1.6 for $0.2<\tau<\infty$ gives 0.79 for the residual intensity, and if $\eta=0$ for $0<\tau<0.3$ and 1.6 for $0.3<\tau<\infty$, the residual intensity is 0.75. At the point $\lambda-\lambda_0=30$ A, $\eta=0$ for $0<\tau<0.2$ and 0.64 for $0.2<\tau<\infty$ gives a residual intensity of 0.90, and if $\eta=0$ for $0<\tau<0.3$ and 0.64 for $0.3<\tau<\infty$, the value is 0.86. The results are not very sensitive with respect to the critical value of τ. The means, $I=0.75$ at $\lambda-\lambda_0=20$ A, and $I=0.88$ at $\lambda-\lambda_0=30$ A, agree well with the computation made above for a variable coefficient of absorption.

The following conclusions may be drawn from Figure 10:

1. Normal pressures, as given by Milne from the ionization theory, produce appreciable Stark effect and modify the contour of $H\gamma$ very markedly.

2. The total energy absorbed in the line increases with Stark effect.[2]

3. The general character of the computed line, as deformed by Stark effect, resembles the observed contours and approaches the exponential curve more closely than does the undeformed line.

4. If we had taken, in the undeformed line, a central intensity closer to zero, we should have found very little flattening in the center of the deformed contour. In fact, if in the undeformed line

[1] *The Internal Constitution of the Stars* (German ed.), p. 425, formulae 236.2 and 236.3, 1928.

[2] This possibility was suggested to us in conversations with Dr. Th. Dunham, Jr., Miss Payne, and Miss Anger. It is also evident on general grounds. Consider two absorption coefficients, σ_1 and σ_2, and let the one be distributed over a narrow region of λ while the other is flat and broad. We have from formula (11) $\int\sigma_1 d\lambda=\int\sigma_2 d\lambda$. But the contours are given by $I_1=f(\sigma_1)$ and $I_2=f(\sigma_2)$. It is clear that in general $\int I_1 d\lambda\neq\int I_2 d\lambda$.

$I_\nu = 0$, for $\lambda - \lambda_0 = 0$ and for $\tau = 0$, then $I_\nu = 0$ for the deformed contour also. This is due to the low pressures of the gases at the top of the reversing layer, and they satisfactorily account for the great number of Balmer lines observed in stellar spectra.

5. Since the Stark effect increases with the serial number we should expect that the total absorptions of successive members of the Balmer series would not decline as rapidly as do the intensities of the emission lines in the laboratory spectra.[1] In fact, we may even have an increase in total absorption with serial number.

6. In the dwarf stars the spread of the wings is greater, and the total absorptions should be greater, than in the giants.

7. A comparison of the observed contours of $H\delta$ in the spectra of ι Herculis and of 67 Ophiuchi with the theoretical ones of Figure 10 shows that it is not possible to convert the line of 67 Ophiuchi into that of ι Herculis by Stark effect alone. In addition to Stark effect there must be present another cause which changes the total absorption. It is possible that the departure from thermodynamic equilibrium discovered by W. S. Adams and H. N. Russell does not equally affect giants and dwarfs.

8. In Figures 8 and 9 we have shown the observed contours in two pairs of stars, the one being a supergiant and the other a normal star or a dwarf. The total absorptions in each pair are approximately equal in order to eliminate the phenomenon mentioned in (7). It will be seen that the contours of both pairs resemble those derived theoretically (except for the scale in abscissa).

9. Since the total absorption increases with the Stark effect, this factor will have to be considered in comparing the observed relative intensities of the hydrogen lines with a theory of thermal ionization.[2] Stars of the same atmospheric density but of different temperatures will show an increase of total absorption with the temperature since there are a larger number of ions (greater Stark effect) at the higher temperatures.

YERKES OBSERVATORY
October 1930

[1] This effect, combined with the one described in our earlier paper on *Si* III lines (*Astrophysical Journal*, **72**, 267, 1930), may give a distribution of intensities wholly different from that observed in emission.

[2] Elvey, *op. cit.*, **71**, 191, 1930.

ON THE VARIATION OF THE INTENSITIES OF SPECTRAL LINES IN α^2 CANUM VENATICORUM

By A. MARKOV

ABSTRACT

The *results obtained by various observers have been compared.* The following conclusions may be drawn: (1) Of 147 spectral *lines* 100 *vary in intensity*, while 37 appear to be constant within the precision of the estimates. The constant lines are among the faintest known in this star. (2) *The variable lines may be grouped into three classes:* class I has maximum intensity of the line when the brightness of the star is at minimum; class II comes to maximum at the same time when the light reaches maximum; and class III shows two maxima. There are comparatively few lines of class III. (3) It is probable that the double maximum of the lines of class III is produced by the effect of blending of two lines of classes I and II, respectively. (4) Estimates of the intensity of certain lines, e.g., 4128 and 4131, suggest that the amplitude of the variation as well as the mean intensity change from one epoch to the next. This may be due to variations in the intensities of long period. (5) It is probable that the *period* of the variation in line intensity *has decreased* since it was discovered by Belopolsky.

A large number of the more conspicuous faint lines of α^2 Canum Venaticorum are known to vary in intensity with a period of 5.5 days. However, the total amplitude of this variation is so small that it is difficult to assign to it a numerical value, or to measure it accurately. For this reason I have attempted to compare the results obtained by various observers.

My own work on this star was started in 1926 from plates taken by A. A. Belopolsky at Pulkovo. The results were in good agreement with the earlier estimates of Belopolsky[1] and of C. C. Kiess.[2] The more recent results of Miss C. J. Anger[3] are also in excellent agreement with the Pulkovo observations. However, I was able by my method of estimates to note certain facts which were not recorded by Miss Anger, probably on account of the smaller dispersion of her spectrograph (one prism as against three at Pulkovo). In addition to the changes of short period there seem to exist variations in the intensity of certain lines having a long period.

The intensities of the lines, or rather their conspicuousnesses, have

[1] *Bulletin de l'Observatoire Central de Pulkovo*, No. 70, 1915; *Bulletin de l'Académie Impériale de St. Petersbourg*, No. 12, 1913.

[2] *Publications of the Observatory of the University of Michigan*, **3**, 1923.

[3] *Astrophysical Journal*, **70**, 114, 1929.

been estimated by the various observers at Pulkovo in different ways. Belopolsky used no comparison scale and depended on a mental scale. He denoted a strong line by 5 and its absence by 0 (on spectrograms of 1913–1915). V. A. Rossovskaya proceeded in very much the same manner (plates of 1915–1918). B. P. Gerasimovič compared the stellar lines to those of a constant scale (plates of 1915–1916). This I was able to ascertain from his original records.

My own determinations of intensity cover the years 1913, 1914, 1919, and 1920. At first I estimated on a certain scale the intensities

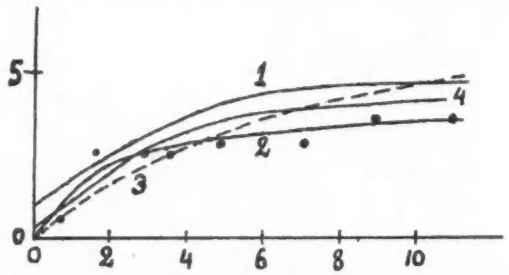


FIG. 1.—Relation between the estimates of Markov (abscissa) and: (1) Belopolsky, (2) Gerasimovič, (3) Miss Anger (difference in the intensity expressed in stellar magnitudes), (4) difference in the density of the continuous spectrum and the line obtained at Pulkovo.

of a selected list of lines marked by Belopolsky as variable. To these I gradually added other lines which also appeared to vary. However, I soon found that better results could be obtained by comparing the lines on each plate to a standard spectrogram of the same star serving as a scale. The most conspicuous line on the standard scale was arbitrarily assigned an intensity of 10. Absence of a line was denoted by 0. The intermediate lines of the scale were calibrated by direct estimates. All estimates were made by the method of E. C. Pickering, which I have previously used in similar photometric problems.[1]

In order to obtain results independent of the scale I also estimated the intensities of all lines visible in the eyepiece of the Zeiss spectrocomparator against λ 4131 as a standard. The curves which I ob-

[1] Determination of the brightness of certain details on the moon: *Astronomische Nachrichten*, **221**, 65, 1924.

tained agree very well with those of Belopolsky. The line λ 4128 due to *Si* II was found to vary with respect to λ 4131 also due to *Si* II.

Finally, I have also measured with the Hartmann microphotometer the difference in the photographic density of certain lines and of the adjoining portions of the continuous spectrum. Figure 1 shows the relationship between my comparisons made with the scale (abscissa) and the determinations of the other observers, including those of Miss Anger obtained with the Moll microphotometer.[1]

From duplicate estimates of the same lines on a single plate I found that the precision of my estimates is ±0.7 divisions of my scale. However, the evaluation of line intensities from different plates taken in close succession yields a larger probable error. This must be due to differences in the photographic density of the individual plates. In order to minimize this source of error, I have given half-weight to all plates taken with exposure times greatly differing from the average, which was about one hour.

The results for some of the lines are shown in Figures 2, 3, and 4. The lines are here divided into three classes: Those of Figure 2 show a pronounced minimum near phase 2–3 days, while those of Figure 3 have a maximum not far from this phase; finally, those of Figure 4 show two maxima and two minima. For comparison I have also included in the diagram several curves obtained from the estimates of Gerasimovič. They agree well, in shape and amplitude, with those obtained from my estimates. The following lines belong to class I: 4003, 4128, 4129 (?), 4131, 4174 (?), 4201, 4234, 4261 (?), 4297 (?), 4304 (?), 4326, 4353, 4404 (?), 4416, and 4481. Only one of these lines completely disappears, according to Belopolsky. To the second class belong the following lines: 4130, 4132, 4290, 4168 (?), 4483. The following lines were found constant within the precision of my estimates: 4010, 4033, 4044, 4076, 4078, 4125, 4165, 4171, 4172, 4178, 4414, 4415. The line at 4013 seems to vary, but it is difficult to fix the class of variation.

Figure 5 shows the appearance of a portion of the spectrum from about λ 4120 to λ 4135 at four phases. It will be seen that the relative intensities are very different.

[1] Curve 3 in Fig. 1 is based upon material published by Miss Anger, as well as upon unpublished data which Miss Anger has kindly sent to me at my request.

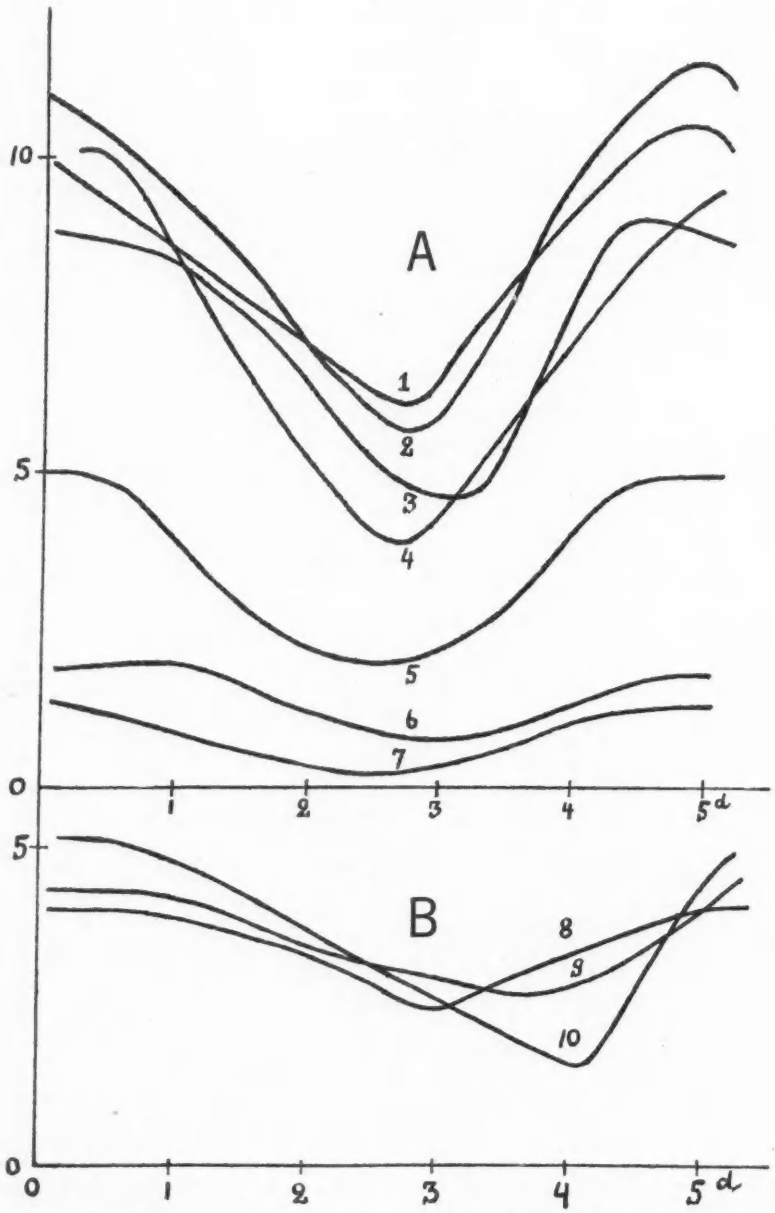


FIG. 2.—Intensity variations for lines of first class, by Markov (*A*) and by Gerasimovič (*B*): (1) λ 4128; (2) λ 4131; (3) λ 4481; (4) and (8) λ 4233; (5) λ 4201; (6) λ 4261; (7) λ 4129; (9) λ 4123; (10) λ 4253.

In Figure 6 I have shown the ratio of the intensity of λ 4128 to λ 4132, obtained without scale and with scale. It will be seen that the curve obtained from the plates of 1923–1914 differs from that of

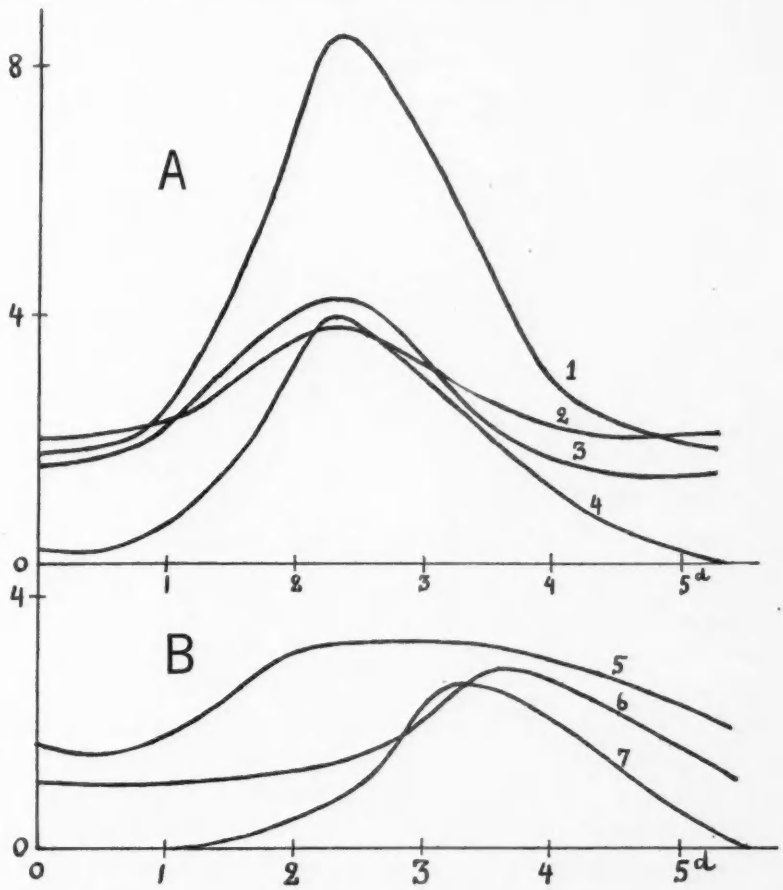


FIG. 3.—Intensity variations for lines of second class, by Markov (*A*) and by Gerasimovič (*B*): (1) λ 4205; (2) and (6) λ 4290; (3) and (5) λ 4132; (4) and (7) λ 4130.

1919–1920. If this is real, it indicates a secular change in the relative intensities of these two lines. This point should further be tested by means of accurate spectrophotometric measurements. I have also the impression that in June, 1920, the following lines were more intense than usual: 4049, 4128, 4131, 4133, 4165, 4168, 4172, 4234,

4261, 4290, 4297, 4304, 4326, 4353, 4384, 4415, 4408, 4481, while no change in the intensity was noted for the following lines: 4123, 4125, 4129, 4130, 4174, 4178, 4201, 4205, 4414, 4404, 4483. The lines 4261, 4396, 4359, 4384, 4405, 4415, were greatly widened at the same time. The unusual character of this star is further evidenced by the discovery of bright lines in 1927 and 1928 which were not formerly noticed in this star. This discovery was due to Belopolsky.

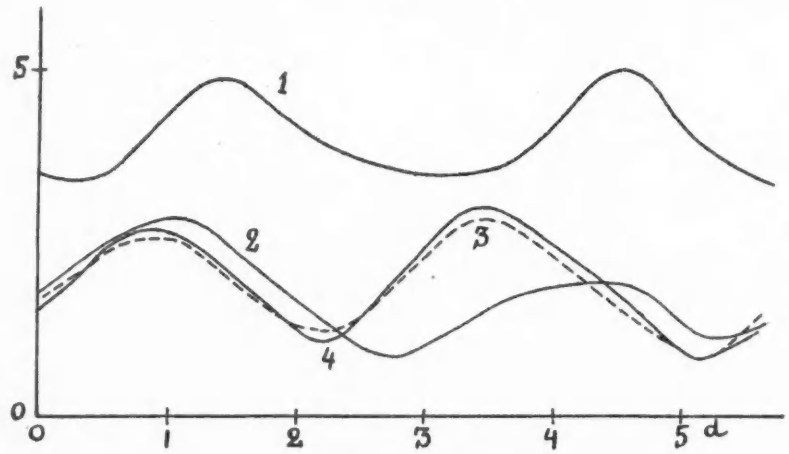


FIG. 4.—Intensity variations for lines of class III: (1) λ 4077 (Gerasimovič); (2) λ 4077 (Markov); (3) λ 4446 (Gerasimovič); (4) λ 4356 (Gerasimovič).

The lines of class III with double maximum are probably due to the effect of blending of a line of class I with a line of class II. That this is really the correct explanation may be seen from Miss Anger's curve for the line 4131. This is a line of class I, but on her diagram there is a second diffuse maximum corresponding in position to the place where the lines of class II usually reach maximum. It is evidently due to the fact that on her plates λ 4130, belonging to class II, was not completely resolved from λ 4131, thus accounting for the double maximum. It is very probable that a similar explanation holds in all cases where double maxima have been observed.

A word may be added concerning the value of the period. In reducing my observations I used Belopolsky's period, 5.4705 days. A comparison of the curves for 1913–1914 and for 1919–1920 shows a

TABLE I

SUMMARY OF LINES OF α^2 CANUM VENATICORUM ACCORDING TO TYPE OF VARIABILITY

CLASS	LINES DEFINITELY BELONGING TO THE GIVEN TYPE (ITALICIZED LINES ARE ACCEPTED BY ALL OBSERVERS)	DOUBTFUL LINES	NUMBER	
			Definite	Doubtful
I (λ4201)	3848.90 *Tb* (5), 3853.9 (5), 3942.35 *Tb* (5), 3954.21 *Tb* (5), 3977.0 *Tb* (5,) 3982.1 (5), *4002.73 Tb* (5), 4033.20 *Tb* (5), 4055.2 (7), 4075.76 (2), 4078.9 (7), *4122.92* (2), *4128.25 Si II, V* (2, 5),* *4131.27 Si II, Ma* (2.5),* *4201.16 Tb* (5), *4232.98 Tb* (5) *Fe* (2), *4260.64 Fe* (2), *4242.5 Tb* (5), 4278.70 *Tb* (5), 4284.06 (3), *4303.46* (2), *4326 Tb* (5), 4352.06 *Ma, Cr* (2), *4383.72 Fe* (2), *4403.35* (2), 4416.99 (7), 4480 (3), *4481.40 Ma** (2), 4530 (3), 4555.4 (7), 4621.6 (5), 4558.6 *Cr, La* (5)	4072.94 (2), 4129 (6), 4179.1 (7), 4294 (3)	34	4
II (λ 4130)	3788.8 (5), 3819.80 *Eu* (5), 3933.7 *Ca* (7), 3978.60 *Eu* (5), 3898.90 *Eu* (5), 3907.29 *Eu* (5), 3925.3 (5), 3930.66 *Eu* (2), 3945.9 bright (7), 3958.0 (5), 3972.06 *Eu* (5), 3988.9 *Ds* (7), 3991.4 *Ds* (7), 4046.88 (7), 4065.4 bright (7), 4068.3 bright (7), 4085.84 *Eu* (5), 4092 (3), 4099.3 (5), 4117 (3), *4129.93 Eu* (2), *4132.4* (2), 4134.7 bright (7), 4143 (3), 4167.41 (7), 4181 (3), 4185 (3), *4205.20 Eu* (5), 4207 (3), 4213 (3), 4221.6 bright (7), 4261.9 (7), 4269.3 (7), 4288.42 (3), 4290.31 *Ma* (2), *Ti* (5), 4330 (3), 4386.97 (3), 4407.8 (7), 4435.78 *Eu* (2), 4444 (3), 4445 (3), 4448.2 *Te* (5), 4483 (6), 4522.6 *Eu* (7), 4592.6 *Eu, Te* (5), 4912.5 *Eu* (5)	4050.3 *La, Ds* (I type), 4144 (3), 4151 (3), 4293 (3), 4450.4 (3)	46	5
III	*4077.06* (2), 4356 (3), 4446 (3)	4134 (3), 4230 (3)	3	2
Variable	4044 (3), 4088 (3), 4407.8 (II or III type, 5), 4415 (3)	4157 (II type? 3), 4253 (I type? 3)	4	2
Constant	4000.4 *Ds* (5), 4019.2 bright (7), 4023.2 bright (7), 4024.6 *Ti* (7), 4046.4 (7), 4049.25 (8), 4058 (3), 4062.0 *Tb* (5), 4063.49*b* (5), 4063.86 *Fe* (5), 4071 (3),	4073 (3), 4171.05 (II type? 2), 4173.72 *Fe*		

* Lines with a possible secular change in the character of their periodic variation. The numbers in brackets indicate the different observers to whom the wave-length, the identification, and the type of variability are due. The notation is as follows: (1) A. Belopolsky, 1913; (2) A. Belopolsky, 1915; (3) B. Gerasimovič; (4) V. Rossovskaya; (5) C. C. Kiess; (6) Markov; (7) C. Anger.

TABLE I—*Continued*

Class	Lines Definitely Belonging to the Given Type (Italicized Lines Are Accepted by All Observers)	Doubtful Lines	Number	
			Definite	Doubtful
Constant	4077.9 *Yt*, *Ds* (5), 4111 (3), *4124.98 Yt* (5), 4146.2 *Ds* (5), *4163.81 Fe* (5), 4168.6 bright (7), *4172.19 Tb* (2), 4177.94 (2), 4187.4 *La* (5), 4190.8 (7), 4214 (3), 4228 (3), 4272 (3), 4321 (3), 4375 (3), 4395 (3), 4382.4 bright (7), 4400 (3), *4404.95 Fe* (2), 4414.5 (5), 4471.14 (3), 4473.29 (3), 4493.5 (7), 4515.5 (7), 4520.2 (7), 4534.30 *Tb* (5)	(II–III type? 2), 4308 (III type? 3), 4176.6 (3), 4368 (II type? 3), 4370 (II type? 3), 4386 (I type? 3), 4456 (III type? 3), 4462 (II type? 3)	37	10
Total			124	23

TABLE II

Variability of Lines in Different Elements

Element	Lines of Class I	Lines of Class II	Constant Lines	Variable Lines	Total
Eu		13			13
Tb	11		2*		13
Ma	3	1			4
Si	2†		3†		5
Fe	3		3		6
Ca		1			1
V	1†				1
Cr	2				2
Ds		2	2		4
Ti		3			3
Bright		5	4		9

When two different elements have been assigned to the same spectrum line without expressing a preference, this line is entered twice in Table II.

* 4172.19 *Tb* and 4534.30 *Tb*.

† 4128 and 4131 of *Si* II. Kiess points to the existence of three *Si* lines in the ultra-violet section of the spectrum which he does not consider to be variable. Belopolsky identifies *Ma* and *V* with the two first *Si* lines.

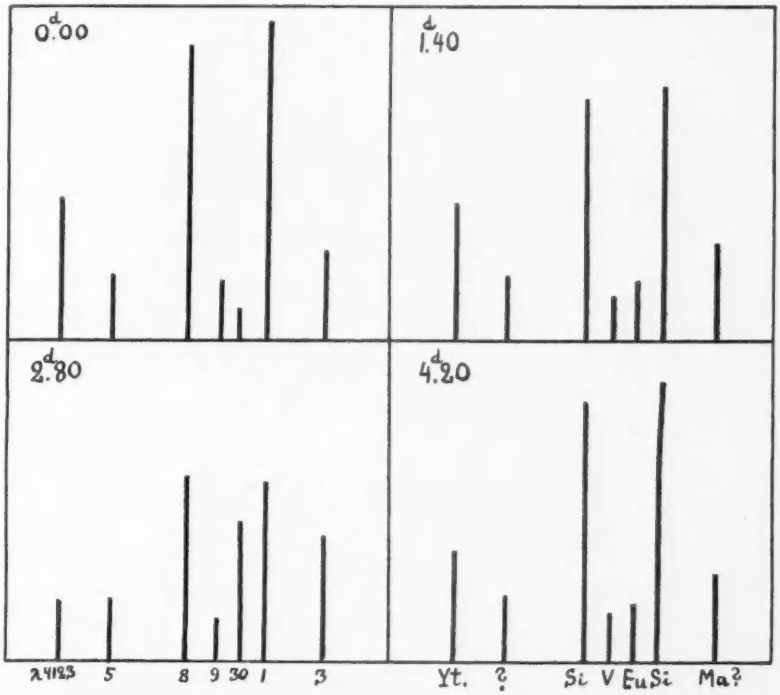


FIG. 5.—Relative intensities of lines in spectrum of α^2 Canum Venaticorum between λ 4123 and λ 4133, at four different phases.

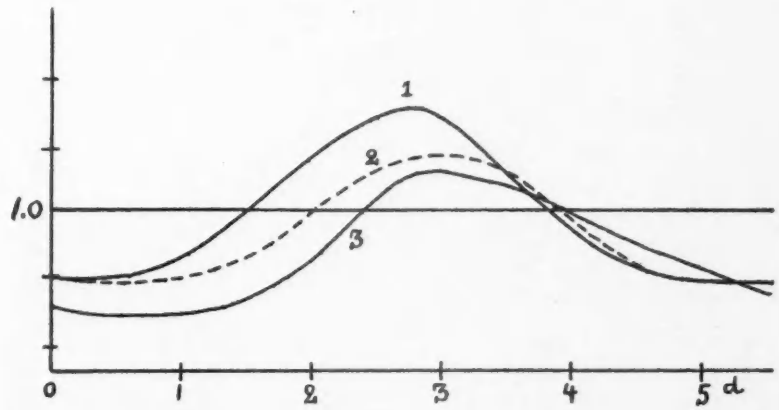


FIG. 6.—Ratio of intensity of λ 4128 to λ 4131 for: (1) 1919–1920 without scale (2) 1919–1920 with scale, (3) 1913–1914.

slight shift, suggesting a change in the period amounting to -0.0010 ± 0.0005 day. A similar correction to the period is obtained by comparing my observations with those of Gerasimovič. From eight lines I find the following correction: -0.0010 ± 0.0005 day. If we assume that this is due to a real change in the period, we obtain by extrapolation for 1927–1928 a period of 5.4678 ± 0.0008 days. This agrees well with the latest value given by Belopolsky, 5.4687 days.

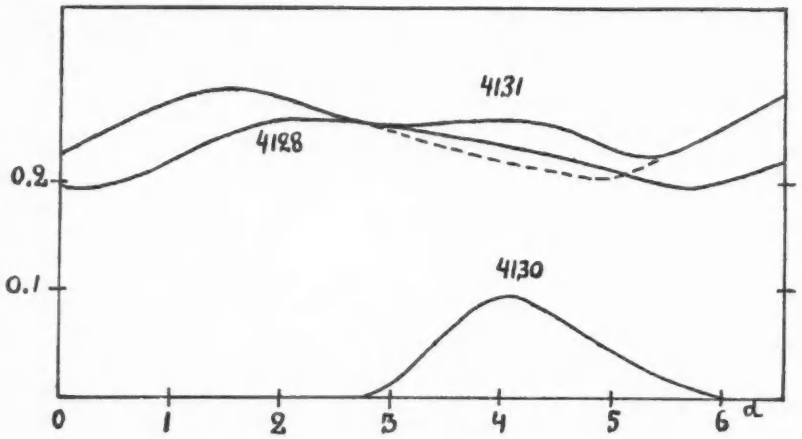


FIG. 7.—Effect of blending upon the curve of intensities for λ 4131

In Table I, I have given a summary of all lines observed by various observers in α^2 Canum Venaticorum, according to the class of variation shown by each. It was possible to assign 124 lines to a definite class, while 23 lines remained doubtful. The number of lines definitely attributed to class I is 34, while the number attributed to class II is 46. In Table II, I have arranged the lines according to element. It will be seen that all lines attributed to europium belong to class II, while all lines attributed to terbium are of class I, except two which are regarded as constant.

PULKOVO OBSERVATORY

MAGNITUDES OF SOUTHERN COMPARISON STARS FOR EROS[1]

By F. H. SEARES, B. W. SITTERLY, AND MARY C. JOYNER

ABSTRACT

Magnitudes, photographic and *photovisual*, have been determined for *300 southern comparison stars for Eros* (Table IV, mag. 7–10) and *74 bright extra stars* (Table V, mag. 5–7.) Scales and zero points were established by polar comparisons and strengthened by connecting plates. Screen exposures give added weight to the brighter Eros stars and provide data for the extra stars.

Mean color index and spectrum.—The mean color index for Ao stars (Harvard spectra) is −0.14 mag. Colors for F, G, and K stars are less than those found by King, Parkhurst, and Seares (Table XI). The presence of dwarfs may account for part but not all of the differences.

Comparison with Yerkes measures.—One hundred and twelve of the stars have been observed by Ross and Zug. Agreement in *zero points* is *good*, but *scale divergence is appreciable* for both photographic and photovisual magnitudes (Table VII). A large accidental difference $MW - V = \pm 0.19$ for photographic magnitudes was traced to a progressive change in zero points. Two independent series of *MW* polar comparisons are accordant (Table IX), while comparisons of color indices for stars of the same spectral type in different fields indicate that the discordance is to be apportioned between *MW* and *V* in the ratio of 3 to 7 (Table X). This assumes the types to be free from error.

Comparisons with Harvard magnitudes.—Provisional formulae for reduction to the international system (Tables XII, XIII) were derived for visual measures by Pickering, Wendell, and Bailey in *H.A.* **14, 24, 44, 45, 46, 70,** and **74**. The color coefficient varies with the observer, instrument, and apparent magnitude of the star. Comparisons with magnitudes in the *Draper Catalogue* which were derived from *Durchmusterung* catalogues—the *SDM*, *CD*, *CPD* (Table XV)—show no scale divergence and only small zero-point differences, in striking contrast with the 34 per cent divergence found by Ross and Zug for the *BD*.

1. THE OBSERVATIONS

The International Commission on Solar Parallax asked the Commission on Stellar Magnitudes to provide for the determination of the magnitudes and colors of the comparison stars[2] of Eros for the opposition of 1930–1931. Dr. F. E. Ross and Mr. R. S. Zug of the Yerkes Observatory consented to undertake the measurement of the northern stars and to extend their observations as far south as observing conditions would permit; the limit actually reached by them[3] is −17°. The stars south of the Equator have been observed with the 10-inch Cooke triplet at Mount Wilson.

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 415.

[2] Kopff, *Astronomische Nachrichten*, **224**, 387; **226**, 33, 1925.

[3] *Astronomische Nachrichten*, **239**, 289, 1930.

The Mount Wilson photographs, all taken by Sitterly, consist of three series:

A. Polar comparisons, 15 photographic and 15 photovisual (Table I); 2 exposures on the field, 2 on the pole, arranged in a symmetrical order. The spacing of the centers is such that each plate includes about half the stars shown on each of the adjacent plates.

B. Centers and exposures as in A, with a wire-gauze screen absorbing 3.01 mag. in front of the objective.

TABLE I

FIELDS AND PHOTOGRAPHS FOR SERIES A AND B

FIELD	α 1930	δ 1930	SERIES A		SERIES B	
			Pg.	Pv.	Pg.	Pv.
1.....	10^h24^m2	$-2°36'$	April 19	April 19	April 19	April 19
2.....	21.7	5 6	18	March 29	20	25
3.....	17.7	8 14	March 26	26	24*	22*
4.....	13.5	11 2	April 18	April 18	22	22
5.....	8.8	13 51	17	17	20	20
6.....	4.7	16 46	25	1	22	22
7.....	9 56.7	18 58	1	25	20	20
8.....	51.6	22 8	25	March 27	March 27	March 27
9.....	45.5	24 16	25	26	25	April 20
10.....	34.9	26 38	March 24	24	24	March 24
11.....	46.0	25 24	25	25	25	April 19
12.....	57.1	24 46	27	27	27	March 28
13.....	10 9.7	23 54	26	26	26	26
14.....	21.3	23 4	April 22	29	28	28
15.....	31.4	-22 23	March 29	29	24	24

* Additional Pg. and Pv. plates were obtained on April 25.

C. Connecting plates, 13 photographic and 13 photovisual (Table II); centered on points midway between the centers for A and B; one exposure with screen, one without. Field 10, at the point where the motion of Eros changes from retrograde to direct, is connected by a single plate with both fields 9 and 11.

The screen exposures strengthen the determination of magnitudes for the brighter comparison stars and extend the investigation to still brighter objects (extra stars) not included in the list for Eros. Several plates of the pole, each with and without screen, determine the screen constant.

Exposures for photographic magnitudes were 2^m on Eastman 40 plates, with the aperture reduced to 6.5 inches to sharpen the images near the edge of the plate (4×5 in.); for photovisual magnitudes,

4^m with the full aperture on Cramer Iso Presto plates through a yellow filter (MW1).

With few exceptions each star occurs on at least two polar comparison plates of series A and two connecting plates of series C. In the region of fields 9, 10, and 11, additional exposures are available, and throughout the list many of the brighter stars are recorded by both screened and unscreened exposures. Finally, the extra stars measured on the screen exposures of C have an independent scale and zero-point determination in the polar comparisons of series B.

TABLE II

PHOTOGRAPHS FOR SERIES C

Fields	Pg.	Pv.	Fields	Pg.	Pv.
1,2	March 28	March 24	8,9	April 1	April 1
2,3	26	26	9,10,11	March 25	March 25
3,4	April 1	April 1	11,12	28	April 17
4,5	17	18	12,13	29	March 29
5,6	March 28	March 28	13,14	April 1	April 1
6,7	April 25	April 19	14,15	March 24	March 24
7,8	1	1			

2. PROVISIONAL REDUCTIONS

The plates were measured (series A and C by Sitterly, B by Miss Joyner) with a low-power microscope provided with a scale of star images placed at the common focus of eyepiece and objective. Scales *a* and *b* were used for series A and C: *a* with large images to the right for the first measurement, *b* with large images to the left for the second. Series B was measured twice with scale *b*, in the direct and the reversed position. For series A and B the order of measurement of polar and field images was inverted for the second series of settings. The adopted scale readings are the means of four estimates, two on each of two images for each star.

Scale readings were transformed into magnitudes by means of the international standards,[1] reduced to the color system of the 10-inch refractor by the formulae[2]

$$Pg_{10} = Pg - 0.19C, \qquad Pv_{10} = Pv,$$

[1] *Transactions of the International Astronomical Union*, **1**, 71, 1922.

[2] *Seares and Humason, Mt. Wilson Contr.*, No. 234; *Astrophysical Journal*, **56**, 84, 1922.

where *Pg*, *Pv*, and the color index *C* refer to the international system (the same as that of the 60-inch reflector). Stars 5–17, 2r–9r, 2s, 4s, and 6s were used for series A. For B, 4 and 1r were also required. Both scale and zero point adopted for the results of this paper depend on the adopted magnitudes of these stars.

The magnitudes of field stars on the polar comparison plates were found by plotting polar magnitudes against scale readings and using the resulting curve as a relation applicable to the field stars on the same plate. Corrections for differential atmospheric extinction between field and pole were obtained from Table XII, page xxxvi, *Mount Wilson Catalogue*.[1] Distance corrections had to be determined from the observations; for the provisional reduction they were neglected.

The provisional magnitudes of field stars from the A plates were used to reduce the plates of series C, each of which gave two curves, derived by plotting scale readings against provisional magnitudes for the two fields covered by the plate. Were it not for instrumental and observational errors, these curves would be identical. As it is, accidental, and especially plate and zero-point, errors affect the curves, and their differences may be used to determine the errors.

Since each curve depends on a separate determination of the zero point, the mean ordinate difference of the curves includes the zero-point errors and should show an accidental distribution of algebraic signs from plate to plate. The ordinate differences actually found from plate pairs 1,2; 2,3; 3,4; etc., of the photographic series, taken in the order field 1 – field 2, field 2 – field 3, etc., down to field 10, were all positive and equal to about 0.4 mag. For fields 10 to 15 the differences were less conspicuous but still systematic. The corresponding differences from the photovisual plates were much smaller and relatively unimportant.

3. PLATE CORRECTIONS

Since the centers of the fields connected by a C plate are in opposite halves of the plate, these large systematic differences in the curves indicated an unusual plate error affecting the Eastman 40 negatives, positive in one half of the plate and negative in the other

[1] *Publications of the Carnegie Institution of Washington*, No. 402, 1930.

half. The cause seems to be an error in the centering of the lenses of the Cooke triplet which seriously affects the photographic rays but has little influence on the light transmitted by the yellow filter.

The determination of the corrections which harmonize the results was greatly facilitated by a manuscript copy of the Yerkes magnitudes kindly supplied by Dr. Ross. Differences $Y-MW$ were formed for each of the hundred-odd stars common to the two series of observations for each of the Mount Wilson plates on which it occurred. These differences were grouped according to magnitude and entered on diagrams at points corresponding to the positions of the stars on the plates. Since the Yerkes magnitudes had been corrected for plate errors, the difference between the value of $Y-MW$ for any point on the plate and its value at the center is the correction to be applied to the Mount Wilson magnitude.

Provisional corrections were thus found for a zone 2° wide, extending across the plate along the path of Eros. These were strengthened and extended to other parts of the plate with the aid of the Mount Wilson measures. Let m_1 and m_2 be the uncorrected magnitudes of the same field star obtained from two polar comparisons. In the absence of other errors, the value of m_1-m_2 will be the difference in the plate errors for the images from which m_1 and m_2 were obtained. If both images are within the region for which provisional corrections are known, m_1-m_2 may be used to revise these corrections; if only one image is within this region, the difference leads to the correction for the outside point. Actually both m_1 and m_2 are affected by zero-point and ordinary accidental errors; but with abundant material the zero-point errors become accidental and only the mean error for the entire series of polar comparisons enters into the final smoothed corrections.

Since the correction depends on the size of the image as well as its position on the plate, separate tables were prepared for the magnitude limits <8, 8–9, 9–10, and >10 for both photographic and photovisual results. The corrections are all asymmetrical with respect to the center, especially so for mag. 8–10 on the plates of the photographic series, for which the corrections range from about $+0.5$ to -0.3 mag. in opposite halves of the plate. The change with brightness near the tenth magnitude is rapid, particularly in the

northern half of the field, and failure to use the exact value of the correction may have introduced a slight discontinuity into the scale of the adopted photographic magnitudes at this point. A similar irregularity may also affect stars near the eighth magnitude, where the transition from one group to the next is not well determined. The corrections to the photovisual results are smaller than those required by the photographic magnitudes, show less asymmetry with respect to the center of the plate, and present some resemblance to what might normally be expected in the way of a distance correction.

4. COLOR COEFFICIENT FOR 10-INCH COOKE TRIPLET

After the correction tables were completed, all the results from the A plates were revised as a preliminary to re-reduction of series C. Before proceeding with C, however, the plots of polar magnitudes used in forming the reduction-curves for series A were examined for evidence bearing on the color coefficient of the 10-inch Cooke triplet. Any error in referring the international standards to the color system of this instrument must show as a systematic difference between the plotted points for red and blue stars in respect to their deviations from the adopted reduction-curve. The mean of these deviations from all the curves was formed for each standard star for both photographic and photovisual results and compared with the aid of the relation

$$x+Cy=\Delta\,,$$

where Δ is the mean deviation and C the color index, and where, further, y is a correction to the color coefficient, 0.19, originally used to refer the standards to the system of the 10-inch telescope, and x a correction to the magnitudes necessitated by the fact that any error in the coefficient used affects the zero point. Incidentally, x also takes care of any systematic error in the position of the curves with respect to the plotted points.

Least-squares solutions gave

x		y
+0.03 mag.	.....	−0.036 (Photographic)
+0.01	.....	−0.026 (Photovisual)

The second value of y is unreliable and has been disregarded; the other corrections were adopted. The revised color coefficient of the

10-inch triplet for photographic magnitudes is therefore 0.19 − 0.036 = 0.154.

For the present, however, the values of x alone need be considered. These were applied to the magnitudes of the field stars as read from

TABLE III

CLOSING ERRORS AND ZERO-POINT CORRECTIONS

(Unit = 0.01 mag.)

FIELD	SERIES A AND C				SERIES B				SERIES C − SERIES B	
	Pg.		Pv.		Pg.		Pv.			
	ϕ	Corr.	ϕ	Corr.	ϕ	Corr.	ϕ	Corr.	Pg.	Pv.
1		(+21)		0		+18		+ 9	− 1	+ 3
	(−24)		+ 2		−37 (5)		−18 (5)			
2		− 3		+ 2		−19		− 9	+15	+12
	− 3		− 3		−12 (1)		−19 (1)			
3		− 6		− 1		0		0	0	+ 4
	+ 2		− 4							
4		− 4		− 5		+ 6		− 3	−13	− 5
	+ 6		−10		−13 (4)		+ 6 (5)			
5		+ 2		−15		− 7		+ 3	+ 2	−17
	+ 4		+14		−32 (2)		−38 (1)			
6		+ 6		− 1		+11		− 2	−12	− 9
	+ 3		0		−23 (5)		+ 3 (4)			
7		+ 9		− 1		−12		+ 1	− 2	+ 7
	− 8		− 5							
8		+ 1		− 6		− 6		+15	− 9	− 8
	+ 5		0		+ 9 (4)		− 6 (2)			
9		+ 6		− 6		+ 3		+ 9	0	− 4
	− 7		+12		− 4 (1)		0 (2)			
10		− 1		+ 6		− 1		+ 9	+ 1	− 3
	0		+ 1		+ 9 (4)		−17 (4)			
11		− 1		+ 7		+ 8		− 8	+ 2	+ 2
	+ 8		− 1		+ 2 (5)		0 (5)			
12		+ 7		+ 6		+10		− 8	− 5	+ 3
	−11		− 2		−12 (4)		− 2 (4)			
13		− 4		+ 4		− 2		−10	− 5	− 4
	− 2		+ 1		−10 (5)		+ 3 (5)			
14		− 6		+ 5		−12		− 7	+ 2	− 8
	− 5		− 2		− 5 (2)		+31 (1)			
15		−11		+ 3		0		0	− 7	+ 8
Sums	+28	+31	+30	+33		+56		+46	+22	+39
	−36	−36	−27	−35		−59		−47	−54	−58
Means		0		0		0		0	− 2	− 1

the reduction-curves of the polar comparison photographs, which were then corrected for plate error and used for the final reduction of series C.

5. ZERO-POINT ADJUSTMENT

The mean differences in the ordinates of the reduction-curves for the C plates were again formed, with the results (unit = 0.01 mag.) shown under the heading ϕ in the second and fourth columns of Table III. The order, as before, is field 1 − field 2, etc., and the numbers themselves, approximately at least, are differences in the zero points for successive fields as determined by the polar comparisons of series A. The comparison for field 1 is clearly defective; but otherwise what in the photographic series first appeared as a persistent positive difference of 0.4 mag. is now reduced to a few hundredths, with a satisfactory alternation of signs. The distribution of signs is not quite accidental, but except for the discordant field 1, the sums of positive and negative differences are in good balance.

Since for measures free from all error ϕ should be zero, the values in Table III may be regarded as closing errors in the equations of condition

$$z_2 - z_1 = 0; \qquad z_3 - z_2 = 0, \text{ etc.},$$

and used to determine corrections to the zero points z_1, z_2, z_3, etc., which will satisfy these conditions. The formulae for the corrections[1] are

$$\begin{aligned} v_1 &= \quad +k_1 \qquad \ldots\ldots, \\ v_2 &= -k_1 + k_2 \qquad v_{14} = -k_{13} + k_{14}, \\ v_3 &= -k_2 + k_3 \qquad v_{15} = -k_{14}, \end{aligned}$$

where the 14 correlates k are defined by

$$\begin{aligned} +2k_1 - k_2 \quad\quad\quad\quad\quad\quad\quad\quad &+\phi_1 = 0, \\ -k_1 + 2k_2 - k_3 \quad\quad\quad\quad\quad\quad &+\phi_2 = 0, \\ \cdots\cdots\cdots\

in which the ϕ's have the successive values entered in Table III. The solution is given by

$$15k_{14}+\Sigma n\phi_n=0\,,$$
$$k_1+k_{14}+\Sigma\phi_n\ =0\,,$$
$$v_{15}=-k_{14},\ v_{14}=v_{15}-\phi_{14},\ \ldots\ .\ v_1=v_2-\phi_1\,,$$

with $v_1=+k_1$ and $\Sigma v_n=0$ as check formulae.

The results are in the third and fifth columns of Table III. The difference ϕ_1 in the second column was disregarded in calculating the corrections. This in effect rejects the zero point given by the photographic polar comparison for field 1. The correction used for this field, +21, reduces the magnitudes to the zero point of field 2.

The adopted corrections were applied to all magnitudes which depend on the polar comparison for the field in question. Thus the correction for field 3, −6, was applied to the results from the A plate for this field, to those from the second reduction-curve for the C plate 2,3, and to those from the first curve for C plate 3,4.

6. MEAN MAGNITUDES FOR COMPARISON STARS

To the results thus collected were added those from the screen images of the comparison stars in so far as they appear on the C plates. These images were treated exactly as the others, except that 3.01, the reduction constant of the screen, was subtracted from the magnitudes read from the curves. The mean of all the results for each star represents the final magnitude, photographic or photovisual, referred to the color system of the 10-inch Cooke triplet.

In forming means the results from a single plate were in general given unit weight. The two or three values obtained from the C plates for each star were combined, since the same mean scale reading was used for both. Results from screen images, however, were treated as independent measures of unit weight.

The mean magnitudes have still to be referred to the international color system. Since the small coefficient found in section 4 has been disregarded, the correction for photovisual results is zero; and since

$$Pg_{10}=Pg-0.154C\,,$$

where C is the color index referred to the international system, the reduction formula for photographic results is

$$Pg = Pg_{10} + 0.182 C_{10} ,$$

where C_{10} is the color index observed with the 10-inch triplet. Final results, all on the international system, are given in Table IV. Spectral types from the *Draper Catalogue* or from *Harvard Bulletin* No. 871 appear in the last column.

7. RESULTS FOR BRIGHT STARS

The 74 bright extra stars measured on the plates of series C were treated in the same way as the comparison stars for Eros. The reduction constant 3.01 mag., used to reduce the screen images, although based on recent data, is practically the same as the value (3.02) found in an earlier investigation.[1] The present result is derived from two polar plates, each with a screen and a full-aperture exposure, and from magnitudes for screened and unscreened images read from the reduction-curves for series C. The separate values are

	Const.	Wt.
N.P. 1930, July 7, Pl. I	3.00 mag.	2
N.P. 1930, July 7, Pl. II	3.01	1
Series C, pg. curves	3.02	3
Series C, pv. curves	3.00	1
Mean	3.01	

The magnitudes of the bright stars derived from series C were supplemented by values from the entirely independent polar comparisons of series B. The number of images on these plates is much less than in the case of A, the reduction-curves are less clearly defined, and the precision lower. A partial check on the zero points, similar to that used for series A and C, is shown in Table III. For series B, however, the values of ϕ are the means of the differences in the individual magnitudes for the same star derived from overlapping polar comparisons, the number of stars in each mean being indicated by the figures in parentheses. Values of ϕ based on 4 or 5 stars are usually satisfactory and are the basis for the zero-point

[1] Seares and Humason, *loc. cit.*

TABLE IV

MAGNITUDES OF COMPARISON STARS FOR EROS

Kopff No.	Pg. Mag.	Pv. Mag.	C.I.	No. Plates	*HD* Sp.
420	11.22	10.10	1.12	2,2	Ko
421	10.24	10.00	0.24	2,2	A
422	9.98	8.96	1.02	2,2	G5
423	9.54	9.50	0.04	2,2	A5
424	9.07	7.29	1.78	2,3	K5
425	9.02	7.78	1.24	3,2	Ko
426	9.23	7.42	1.81	3,3	K2
427	9.20	8.94	0.26	4,3	F2
428	9.85	8.79	1.06	4,4	Ko
429	9.04	8.47	0.57	4,4	G5
430	10.04	9.64	0.40	5,4	F8
431	10.60	10.20	0.40	3,3	F5
432	11.07	9.63	1.44	4,4	K5
433	11.68	10.29	1.39	4,4	M
434	9.76	9.67	0.09	5,4	F2
435	8.77	8.68	0.09	4,4	Ao
436	10.93	10.65	0.28	3,2	F8?
437	8.82	8.52	0.30	5,4	F8
438	11.06	10.17	0.89	4,3	K2
439	8.00	8.06	−0.06	5,5	B9
440	10.84	9.99	0.85	5,4	Ko
441	10.51	10.23	0.28	4,5	G5
442	10.32	9.94	0.38	4,4	F5
443	10.68	9.73	0.95	3,3	.....
444	9.53	9.71	−0.18	5,5	A7
445	10.13	9.35	0.78	5,5	Ko
446	9.09	8.76	0.33	5,5	F8
447	8.67	7.36	1.31	4,3	Ko
448	8.66	8.33	0.33	5,5	F5
449	9.75	9.02	0.73	5,5	K5
450	10.79	10.47	0.32	6,5	.....
451	9.47	8.69	0.78	6,6	Ko
452	10.11	9.35	0.76	6,6	Ko
453	10.18	9.15	1.03	6,6	G5
454	9.04	8.07	0.97	5,5	Ko
455	10.88	10.03	0.85	3,3	.....
456	8.99	8.56	0.43	5,6	Go
457	10.93	9.46	1.47	3,3	K5
458	10.79	9.40	1.39	6,6	Mo
459	9.89	9.05	0.84	6,6	Ko
460	9.68	9.54	0.14	5,4	F5
461	10.06	9.30	0.76	8,7	Ko
462	9.07	8.30	0.77	7,7	Ko
463	10.47	9.13	1.34	3,3	K2
464	9.21	8.45	0.76	7,7	Ko

TABLE IV—*Continued*

Kopff No.	Pg. Mag.	Pv. Mag.	C.I.	No. Plates	*HD* Sp.
465	10.32	9.27	1.05	6,5	K
466	10.43	9.15	1.28	4,3	K_5
467	10.01	9.57	0.44	3,3	F8
468	10.76	9.83	0.93	7,7	G_5
469	9.57	8.71	0.86	7,6	K_2
470	9.63	8.29	1.34	6,7	K_2
471	8.95	9.42	−0.47	8,8	B8
472	11.56	10.51	1.05	6,5	K_5
473	8.71	8.73	−0.02	3,3	A_5
474	8.46	8.44	0.02	5,4	Fo
475	8.74	9.00	−0.26	6,6	Ao
476	9.93	9.28	0.65	4,5	G_5
477	9.81	9.46	0.35	6,5	Ko
478	10.02	9.90	0.12	6,6	Go
479	10.75	10.34	0.41	7,7	Go
480	9.24	9.07	0.17	7,7	F_5
481	8.69	7.97	0.72	8,7	G_5
482	10.10	9.93	0.17	3,3	A_2
483	9.68	9.29	0.39	3,3	Go
484	9.61	9.22	0.39	3,3	G_5
485	9.73	8.43	1.30	5,5	K_5
486	9.65	8.35	1.30	7,7	K_5
487	9.95	9.49	0.46	4,3	G_5
488	9.89	8.90	0.99	3,3	G_5
489	9.22	8.06	1.16	6,6	K_2
490	9.17	8.59	0.58	4,3	G_5
491	8.79	8.51	0.28	6,5	Go
492	10.33	9.98	0.35	6,6	F_5
493	10.33	10.00	0.33	4,3	G
494	10.25	9.17	1.08	3,3	Ko
495	9.95	9.17	0.78	3,4	G_5
496	10.76	9.21	1.55	4,4	K_5
497	8.92	7.93	0.99	3,3	Ko
498	10.10	9.97	0.13	8,8	F_5
499	8.56	8.04	0.52	11,8	Go
500	9.60	9.69	−0.09	7,6	Fo
501	9.96	9.50	0.46	3,3	Ko
502	8.64	8.36	0.28	8,7	F8
503	10.61	9.64	0.97	4,3	Ko
504	11.61	10.36	1.25	4,3	.....
505	10.10	9.78	0.32	3,3	F8
506	10.40	9.97	0.43	7,7	Go
507	9.98	9.28	0.70	8,7	Ko
508	10.16	9.30	0.86	4,4	Ko
509	10.15	9.65	0.50	3,3	F8

TABLE IV—*Continued*

Kopff No.	Pg. Mag.	Pv. Mag.	C.I.	No. Plates	*HD* Sp.
510	10.72	9.18	1.54	3,3	K7
511	8.43	8.11	0.32	11,7	F5
512	9.18	9.30	−0.12	7,7	Fo
513	10.88	10.62	0.26	4,2	.....
514	9.57	9.30	0.27	3,4	F8
515	10.14	9.68	0.46	3,3	Fo
516	10.94	10.05	0.89	6,6	K5
517	9.20	9.22	−0.02	7,6	Fo
518	8.50	8.67	−0.17	4,3	Ao
519	10.42	10.18	0.24	4,3	F2
520	11.62	10.51	1.11	5,3	.....
521	9.03	8.94	0.09	3,3	A5
522	9.26	9.30	−0.04	4,3	A2
523	8.85	8.60	0.25	3,3	Ao
524	10.24	9.53	0.71	5,5	Ko
525	9.44	9.20	0.24	3,3	F5
526	8.83	8.89	−0.06	7,6	Fo
527	10.34	10.23	0.11	4,3	Fo
528	9.17	7.54	1.63	3,3	K2
529	10.48	9.16	1.32	4,4	K5
530	10.64	9.67	0.97	4,4	K5
531	8.13	7.93	0.20	4,3	Ao
532	10.78	9.50	1.28	3,3	K2
533	10.40	9.45	0.95	6,6	K2
534	10.51	9.52	0.99	4,3	Ko
534c	10.69	10.29	0.40	4,3	.....
535	10.53	9.50	1.03	4,4	K2
536	9.64	9.60	0.04	4,4	A2
537	9.76	9.55	0.21	3,3	G5
538	10.67	9.95	0.72	4,3	Ko
539	10.01	9.61	0.40	3,3	F5
540	10.69	8.94	1.75	3,3	Mo
541	9.58	9.05	0.53	3,3	G5
542	10.50	9.71	0.79	4,4	.....
543	10.36	9.58	0.78	6,5	F8
544	9.71	8.85	0.86	4,3	Ko
545	10.35	9.32	1.03	5,5	Ko
546	10.00	9.23	0.77	3,3	Ko
547	11.01	9.72	1.29	5,5	K5
548	8.54	7.72	0.82	4,3	G5
549	10.79	10.03	0.76	3,3	F5
550	12.20	10.31	1.89	5,3	F2
551	10.83	9.01	1.82	4,3	M2
552	10.44	10.37	0.07	5,4	F8
553	9.59	9.34	0.25	5,4	F8

TABLE IV—*Continued*

Kopff No.	Pg. Mag.	Pv. Mag.	C.I.	No. Plates	*HD* Sp.
554	8.08	8.14	−0.06	5,4	A0
555	9.49	9.25	0.24	3,3	F8
556	8.10	7.65	0.45	7,4	F5
557	9.02	9.04	−0.02	7,4	F0
558	9.88	9.51	0.37	3,3	G0
559	10.96	9.28	1.68	3,3	K7
560	8.40	7.99	0.41	4,3	F0
561	9.39	8.34	1.05	5,4	K5
562	9.83	9.57	0.26	4,4	G0
563	11.14	9.20	1.94	3,3	M1
564	10.30	9.89	0.41	4,4	F5
565	10.54	9.37	1.17	3,3	K5
566	8.96	7.86	1.10	4,4	K0
567	10.10	9.40	0.70	4,4	K2
568	8.24	7.37	0.87	4,3	G5
569	12.02	10.40	1.62	3,2	.....
570	8.75	8.23	0.52	5,4	G5
571	9.41	8.05	1.36	5,4	Ma
572	9.21	9.26	−0.05	4,4	A5
573	11.68	10.29	1.39	4,4	K0
574	9.27	9.29	−0.02	3,3	A2
575	10.06	9.89	0.17	3,3	F5
576	10.49	9.67	0.82	3,3	K5
577	8.89	8.57	0.32	4,3	F0
578	9.72	10.03	−0.31	4,4	A0
579	9.67	8.37	1.30	3,3	K2
580	9.90	10.23	−0.33	4,4	A0
581	8.84	8.58	0.26	4,4	F8
582	9.52	8.85	0.67	3,3	K2
583	9.32	7.83	1.49	4,3	Ma
584	10.59	10.32	0.27	4,3	F8
585	9.25	8.93	0.32	3,3	F8
586	8.58	8.58	0.00	3,2	F0
587	10.10	9.34	0.76	3,3	G5
588	9.43	9.10	0.33	3,3	K0
589	10.39	9.82	0.57	4,5	G5
590	9.00	8.88	0.12	3,3	F0
591	10.64	10.36	0.28	4,5	G0
592	9.91	9.70	0.21	2,2	F5
593	10.03	9.47	0.56	4,4	K0
594	9.78	9.92	−0.14	3,3	F5
595	9.83	8.99	0.84	3,3	G5
596	10.64	9.97	0.67	4,4	K0
597	9.15	9.14	0.01	4,4	F0
598	9.68	9.76	−0.08	3,3	F0

TABLE IV—*Continued*

Kopff No.	Pg. Mag.	Pv. Mag.	C.I.	No. Plates	*HD* Sp.
599	9.91	9.69	0.22	4,4	F2
600	10.38	8.96	1.42	3,3	Ma
601	10.23	9.98	0.25	3,3	F8
602	11.21	10.50	0.71	4,1	G
603	7.88	7.84	0.04	5,3	Fo
604	10.03	10.07	−0.04	4,4	F2
605	10.72	9.96	0.76	3,3	K2
606	11.04	10.08	0.96	3,3	K5
607	9.98	9.80	0.18	4,4	G5
608	10.43	9.59	0.84	3,3	K2
609	8.63	8.88	−0.25	4,4	A5
610	8.03	7.08	0.95	5,4	G5
611	8.68	8.53	0.15	3,2	F5
612	9.12	8.22	0.90	3,4	Ko
613	9.32	8.48	0.84	3,3	K5
614	10.37	9.97	0.40	5,5	Go
615	10.42	9.53	0.89	3,3	Ko
616	8.38	7.35	1.03	5,7	Ko
617	9.97	10.01	−0.04	5,3	A5
618	9.91	9.59	0.32	3,3	Go
619	9.47	9.60	−0.13	4,4	F8
620	8.34	8.29	0.05	6,5	A3
621	9.70	9.35	0.35	4,4	G5
622	9.62	8.69	0.93	3,3	Ko
623	9.75	9.53	0.22	4,4	Go
624	9.02	8.94	0.08	3,2	F8
625	9.72	8.70	1.02	3,3	K5
626	8.09	8.43	−0.34	4,3	A2
627	8.88	9.27	−0.39	4,4	F2
628	8.68	8.55	0.13	6,5	F5
629	10.15	10.00	0.15	3,3	F8
630	9.36	8.82	0.54	3,3	K2
631	9.31	9.14	0.17	3,3	Fo
632	8.54	7.75	0.79	6,5	Ko
633	9.67	9.41	0.26	4,4	F8
634	9.65	9.54	0.11	4,5	A5
635	8.21	8.06	0.15	6,6	A2
636	10.20	9.23	0.97	4,3	K5
637	9.82	9.22	0.60	3,3	Ko
638	9.49	8.62	0.87	3,3	Ko
639	8.12	6.62	1.50	6,6	K2
640	10.38	10.38	0.00	4,4	Ao
641	10.77	10.44	0.33	4,2	Fo
642	7.60	7.68	−0.08	6,3	A5
643	9.68	8.94	0.74	3,3	Ko

TABLE IV—*Continued*

Kopff No.	Pg. Mag.	Pv. Mag.	C.I.	No. Plates	*HD* Sp.
644	9.40	9.14	0.26	4,3	F5
645	10.29	9.96	0.33	3,3	Go
646	10.56	9.34	1.22	4,4	Ko
647	10.58	9.35	1.23	3,3	K5
648	9.21	8.60	0.61	3,3	Ko
649	10.00	9.09	0.91	3,3	Mo
650	9.42	8.76	0.66	4,4	Ko
651	9.91	9.14	0.77	4,4	Ko
652	10.13	10.09	0.04	4,4	F2
653	9.74	9.18	0.56	5,4	G
654	11.10	10.17	0.93	3,3	K2
655	9.52	9.31	0.21	3,3	G5
656	9.97	9.80	0.17	5,5	F8
657	9.16	9.02	0.14	2,2	F8
658	8.34	7.04	1.30	2,1	G5
659	9.58	8.82	0.76	4,4	K2
660	10.49	9.39	1.10	5,5	K5
661	8.18	7.14	1.04	5,5	Ko
662	8.91	7.94	0.97	5,4	K5
663	9.85	9.01	0.84	5,5	K2
664	7.56	7.73	−0.17	1,1	Ao
665	10.16	10.16	0.00	5,5	Go
666	9.39	9.26	0.13	1,1	Go
667	9.35	8.52	0.83	3,3	K2
668	11.10	9.78	1.32	1,1	K
669	10.36	9.53	0.83	3,3	K5
670	10.34	9.42	0.92	3,3	Ko
671	10.83	10.49	0.34	5,4	Go
672	8.72	7.68	1.04	7,5	Ko
673	10.03	9.84	0.19	1,1	F8
674	9.29	8.24	1.05	3,3	K2
675	9.75	9.53	0.22	2,1	F8
675c	10.88	.....	.....	2,–	.....
676	9.70	9.57	0.13	4,4	Go
677	10.53	9.70	0.83	4,4	K5
679	10.44	9.48	0.96	1,1	Go
680	9.29	8.34	0.95	2,2	K2
681	9.31	9.12	0.19	3,3	F5
684	9.99	8.48	1.51	3,3	K5
685	9.30	9.02	0.28	4,4	G5
686	8.48	8.28	0.20	3,2	F2
688	9.81	9.47	0.34	3,3	Go
689	9.75	8.05	1.70	4,5	Ma
690	9.05	8.85	0.20	4,3	F5
693	9.09	9.11	−0.02	4,4	A5

TABLE IV—*Continued*

Kopff No.	Pg. Mag.	Pv. Mag.	C.I.	No. Plates	*HD* Sp.
693c.....	10.65	10.38	0.27	4,1	.....
696.....	9.67	8.15	1.52	4,5	K5
697.....	8.70	8.36	0.34	3,3	Go
698.....	9.96	9.46	0.50	4,3	F8
699.....	10.02	9.56	0.46	1,1	F5
703.....	9.27	7.86	1.41	1,1	K2
706.....	9.74	9.65	0.09	3,3	F5
714.....	9.04	8.26	0.78	2,2	Ko
716.....	10.00	9.04	0.96	4,4	Ko
717.....	10.11	9.73	0.38	1,1	F5
720.....	9.12	8.92	0.20	1,1	G5
726.....	9.80	9.78	0.02	4,4	F2
727.....	9.44	9.11	0.33	4,4	G5
728.....	8.94	8.31	0.63	4,3	Ko
737.....	8.78	8.98	−0.20	1,1	F8
749.....	9.14	8.95	0.19	3,3	F5
752.....	9.62	8.37	1.25	3,3	K5
757.....	9.86	9.67	0.19	3,3	F8
769.....	8.37	7.63	0.74	4,4	Ko
774.....	9.13	9.17	−0.04	3,3	A3
782.....	10.05	8.99	1.06	3,2	K2
785.....	9.21	9.02	0.19	3,2	A2
788.....	10.90	10.64	0.26	3,2	.....
796.....	10.06	9.22	0.84	2,2	Ko
798.....	10.34	9.56	0.78	2,2	G5
813.....	10.05	9.34	0.71	2,2	Ko
818.....	9.18	8.93	0.25	2,2	F8
819.....	7.86	7.22	0.64	3,3	Ko
820.....	9.61	9.00	0.61	2,2	Go
821.....	9.29	8.82	0.47	2,2	G5

corrections in the seventh and ninth columns of the table. The corrections for fields 8 to 14 were found by the adjustment formulae of section 5; the others by distributing the better-determined differences equally between the two fields from which they were derived.

Further evidence on the internal agreement is shown by the differences in the last two columns of Table III, which represent the deviation of each B plate from the mean magnitudes for the extra stars found from series C. Since these magnitudes from series C depend on from two to four polar comparisons of series A, the differences in question are largely zero-point errors in the plates of series B. Their average is ±0.06 mag., about that normally to be expected

from single polar comparisons; the uncertainty arising from the small number of images on the B plates has been offset by the corrections applied to the zero points.

The value of these corrections is shown by their effect on the differences. Of the 30 values in the last two columns of Table III, 18 are smaller, 6 larger, and 6 the same as would have been obtained without adjustment for series B. As for the adjustment of series A and C, 17 differences are smaller, 11 larger, and 2 unchanged. The improvement for the photographic magnitudes is marked, 11 differences being smaller and only 4 increased.

The distribution of algebraic signs in the last two columns of Table III excludes the possibility of any large progression in the zero-point differences for series B and C; the general systematic means $(C-B)$ for photographic and photovisual results are, respectively, -0.02 and -0.01 mag.

The mean magnitudes for the bright extra stars derived from both series B and C are given in Table V, which is similar to Table IV.

The average probable error of a magnitude from a single plate, including scale and zero-point errors, is

	Pg.	Pv.
Comparison stars.....	±0.096	±0.081
Extra stars.....	±0.093	±0.090

These mean values were calculated from

$$\text{P.E.} = \frac{0.845\Sigma v}{\sqrt{n(n-1)}} \qquad (n = 2, 3, \ldots .),$$

where Σv is the mean of the sums of residuals without regard to sign for all stars for which the number of residuals n is the same. The agreement for different values of n is excellent.

8. COMPARISON WITH THE MAGNITUDES OF ROSS AND ZUG

The satisfactory internal agreement shown by the preceding discussion becomes much less reassuring when the magnitudes in Table IV are compared with those of Ross and Zug. The differences for 112 stars occurring in both series of measures are given in Table VI. The large discordance for No. 569 seems to arise from a wrong identi-

TABLE V

MAGNITUDES OF BRIGHT EXTRA STARS

HD No.	Pg. Mag.	Pv. Mag.	C.I.	No. Plates	*HD* Sp.
83261.....	6.84	6.85	−0.01	4,2	F2
83332.....	6.87	5.83	1.04	5,3	K0
83953.....	5.08	4.57	0.51	4,3	B2p
84117.....	5.54	4.82	0.72	4,4	G0
84367.....	5.29	4.88	0.41	3,3	F5p
84458.....	7.25	7.27	−0.02	5,4	F0
84709.....	7.33	7.15	0.18	5,5	F2
85043.....	7.02	6.69	0.33	8,6	F0
85725.....	6.86	6.38	0.48	6,3	F8
85859.....	6.06	4.75	1.31	6,5	K0
86266.....	6.35	6.47	−0.12	6,5	A2
86463.....	6.75	6.99	−0.24	7,6	A2
86612.....	5.98	6.32	−0.34	7,7	B5
87019.....	7.71	6.97	0.74	6,4	G5
87318.....	6.65	6.76	−0.11	5,4	A0
87427.....	6.07	5.86	0.21	4,4	F0
87793.....	7.52	7.25	0.27	7,5	F8
87870.....	8.62	7.00	1.62	5,2	Ma
88806.....	7.98	6.38	1.60	6,4	Ma
89828.....	6.32	6.49	−0.17	4,4	A0
89816.....	6.42	6.53	−0.11	4,4	A3
90027.....	6.48	6.94	−0.46	4,5	B9
90197.....	7.36	7.23	0.13	6,6	G0
90955.....	6.95	7.49	−0.54	5,5	A0
90954.....	7.24	7.17	0.07	5,5	F0
91398.....	6.79	7.10	−0.31	2,2	B9
91550.....	6.60	5.28	1.32	2,2	K2
91706.....	6.55	5.84	0.71	2,2	F5
91975.....	8.63	6.82	1.81	3,2	K5
92098.....	8.32	7.21	1.11	3,3	K0
84412.....	7.89	7.12	0.77	3,3	G5
84669.....	8.69	7.09	1.60	2,2	K0
85905.....	5.98	6.33	−0.35	4,4	A2
87810.....	7.25	6.58	0.67	1,1	F2
87130.....	7.11	6.70	0.41	4,3	A0
86626.....	8.05	6.71	1.34	4,3	K2
85951.....	6.45	4.81	1.64	2,3	Ma
87344.....	6.02	6.37	−0.35	3,3	A0
87330.....	6.81	7.40	−0.59	3,3	A0
86030.....	8.43	7.14	1.29	2,2	Ma
87768.....	6.77	6.89	−0.12	4,5	F0
87808.....	7.03	5.52	1.51	5,4	K2
88359.....	8.72	7.22	1.50	5,3	K0
86371.....	6.57	6.88	−0.31	3,1	F0
88346.....	8.74	7.20	1.54	3,3	K2

TABLE V—*Continued*

HD No.	Pg. Mag.	Pv. Mag.	C.I.	No. Plates	*HD* Sp.
88025.....	5.94	6.29	−0.35	3,3	Ao
88549.....	6.81	7.18	−0.37	3,3	Ao
89746.....	6.57	6.52	0.05	2,3	A2
89351.....	7.88	7.08	0.80	5,6	G5
88108.....	8.74	7.11	1.63	5,4	Ma
87096.....	7.34	6.97	0.37	3,3	F8
87504.....	4.47	4.53	−0.06	1,2	B8
88215.....	5.76	5.20	0.56	2,3	Fo
89455.....	6.16	6.03	0.13	3,3	Fo
88284.....	4.71	3.71	1.00	1,3	Ko
88182.....	6.23	6.40	−0.17	3,3	A2
89033.....	6.85	6.12	0.73	3,3	Ko
89945.....	8.63	7.14	1.49	3,2	Ma
89565.....	6.34	6.34	0.00	3,3	F2
88195.....	5.6	6.0	−0.4	2,1	Ao
88333.....	6.74	5.69	1.05	1,2	Ko
89254.....	5.51	5.36	0.15	3,3	Fo
88372.....	5.9	6.4	−0.5	2,2	Ao
90362.....	6.91	5.52	1.39	3,4	K5
90430.....	6.65	6.96	−0.31	3,4	Ao
91318.....	7.41	6.81	0.60	3,3	Ko
89669.....	6.59	6.91	−0.32	4,4	A2
89490.....	6.90	6.33	0.57	4,3	Ko
90485.....	7.37	6.42	0.95	4,3	Ko
90044.....	5.72	5.94	−0.22	3,3	B9
91638.....	6.86	6.69	0.17	3,3	F8
90763.....	5.95	6.03	−0.08	3,3	Ao
90057.....	8.11	6.31	1.80	4,3	K5
90882.....	5.44	5.22	0.22	2,2	B9

fication. The star in Kopff's list is B.D.−15°2990. The presence of the brighter star B.D.−15°2989, 7′ and 5^s Np No. 569, possibly affords an explanation of the Yerkes magnitudes.

Scale divergence and accidental error.—Grouped according to magnitude and reduced to mean values, both with and without regard to algebraic sign, the differences of Table VI give the results in the second and third and the sixth and seventh columns, respectively, of Table VII. The run in the systematic differences represents the divergence in the scales of the Mount Wilson and Yerkes magnitudes, which is appreciable for both photographic and photovisual results. The means for all magnitudes together, −0.03 and 0.00

TABLE VI

COMPARISON WITH THE MAGNITUDES OF ROSS AND ZUG

(Unit=0.01 mag.)

KOPFF NO.	*MW−Y*		KOPFF NO.	*MW−Y*	
	Pg.	Pv.		Pg.	Pv.
522	+ 47	+21	610	+ 9	+16
531	+ 49	+12	611	+ 5	+20
532	+ 35	− 2	613	− 4	+17
534	+ 33	+ 6	615	−15	− 7
538	+ 18	−17	617	− 4	+13
540	+ 33	+17	618	−16	−10
541	+ 27	0	619	−14	− 5
542	+ 23	+15	621	−21	+ 3
548	+ 37	+10	622	− 2	+ 2
549	+ 36	−23	623	−14	+ 5
551	+ 22	0	624	+ 8	+30
554	+ 28	+13	625	−18	+21
555	+ 26	+25	626	−30	+ 7
559	− 2	−16	627	−24	+22
560	+ 31	+ 1	629	−16	+ 1
562	+ 38	+13	630	− 9	+17
563	+ 17	− 4	631	−11	−12
564	+ 3	−26	633	−30	− 9
565	+ 15	0	636	−35	+15
566	+ 9	+ 4	637	−11	− 2
568	+ 52	+ 7	638	− 8	+12
569	(+143)	(−40)	639	− 4	− 5
571	+ 10	+12	641	0	− 6
574	+ 32	+ 8	642	−14	+ 8
575	+ 27	−10	643	−24	−16
576	− 4	+ 2	645	−22	−22
577	+ 15	+ 9	647	−17	−12
579	− 13	+ 2	648	−35	− 6
581	+ 13	+10	649	−25	− 9
583	+ 7	+21	650	−19	+ 8
585	+ 29	+10	651	−13	−15
586	− 9	+12	652	− 7	+22
587	+ 3	− 2	654	−42	− 8
588	− 10	+27	655	−33	− 1
590	+ 3	+ 1	657	−25	−10
592	− 15	+ 4	658	+19	− 8
594	− 13	+10	659	−44	+12
595	− 3	+15	661	−14	−23
598	− 15	+15	662	−12	+ 3
599	+ 4	−25	664	− 7	−20
600	− 13	+ 4	666	−10	+11
601	+ 21	−10	667	−48	− 1
602	+ 3	− 9	668	+21	−11
603	+ 7	+10	669	−12	−27
605	− 10	−12	670	−20	−10
606	− 45	−11	673	−14	+ 4
608	− 12	−14	674	−33	−11
609	− 39	+16	675	−21	−11

TABLE VI—*Continued*

Kopff No.	*MW−Y*		Kopff No.	*MW−Y*	
	Pg.	Pv.		Pg.	Pv.
677.....	−13	−36	697.....	−18	−13
679.....	+18	+ 5	699.....	+23	− 4
680.....	−34	− 7	703.....	−10	− 6
681.....	+ 6	− 5	706.....	−19	+11
684.....	−22	−26	714.....	− 9	− 6
686.....	− 1	+ 7	717.....	+17	+ 5
688.....	−27	−13	720.....	−11	+ 7
690.....	− 6	−16	737.....	−39	+ 3

mag., respectively, are the zero-point differences for the two series of observations.

The photographic magnitudes have all been reduced to the international color system, and, in fact, their differences show no dependence on color. Ross and Zug have assumed that their photovisual magnitudes are also on this system. Relative to the Mount Wilson values, they indicate, however, a small but definite color coefficient:

$$MW - Y = -0.06C .$$

Had the doubtful color correction found in section 4 for the Mount Wilson photovisual magnitudes been applied, the color difference $MW-Y$ would have been larger still. The difference has been disregarded in the remaining comparisons.

The third and seventh columns of Table VII, which include accidental as well as systematic errors, indicate the accordance between Mount Wilson and Yerkes for individual stars. The average of ± 0.11 mag. for the photovisual series is acceptable; but ± 0.19 for the photographic magnitudes is very unsatisfactory.

Table VI shows that this large value originates in a zero-point difference which changes with the position of the stars along the path of Eros. The fields involved, in the order of star numbers, are 7 to 1 of Table I, covering an interval of about 30^m in R.A. and 17° in Dec.

Agreement for Mount Wilson plates.—In searching for the cause of the difference, the first question concerns the agreement of results

for the same star from different Mount Wilson plates. Table VIII gives data for four stars showing large positive and negative differences, but otherwise selected at random. The magnitudes read from the curves include the extinction correction, but both plate and zero-point corrections are given in detail. Bracketed magnitudes depend on the same image on a connecting plate, reduced with different polar comparisons, and are therefore united to obtain a value of unit weight. The last pair for star 531 is from screen images, and allowance must be made for the screen constant of 3.01 mag.

TABLE VII

SUMMARY FOR $MW-Y$

PHOTOGRAPHIC				PHOTOVISUAL			
Yerkes Mag.	Syst. Diff.	Aver. Diff.	No. Stars	Yerkes Mag.	Syst. Diff.	Aver. Diff.	No. Stars
7.5– 8.0	+18	±24	7	<7.5	− 3	±12	5
8.0– 8.5	+ 5	19	7	7.5– 8.0	+ 5	10	11
8.5– 9.0	+12	17	11	8.0– 8.5	+ 6	11	14
9.0– 9.5	− 5	17	19	8.5– 9.0	+ 8	12	18
9.5–10.0	−14	20	26	9.0– 9.5	+ 1	9	27
10.0–10.5	+ 1	20	23	9.5–10.0	− 2	10	25
10.5–11.0	− 7	15	15	>10.0	−16	±16	11
>11.0	−28	±30	3				
Means and totals	− 3	±19	111	Means and totals	0	±11	111

The reduction for color transfers the results to the international color scale.

The zero points depend directly on the polar comparisons of series A for the fields listed in the first column of Table VIII; for example, Nos. 9, 10, and 11 in the case of star 522. In fact, all the comparisons are involved through the adjustment process, which assigns to each field a correction depending on all the observed zero points. The agreement is such that something more than the ordinary accidental error affecting polar comparisons is required to explain the large deviations from the Yerkes results. The plate corrections are large, but not in general strongly systematic for a given star; the accordance which they bring into the results is good testimony of their general validity.

Test of zero points by series B.—A further test is afforded by the independent polar comparisons of series B, which have been used only for the extra stars of Table V and do not enter into the results for the Eros stars. The agreement of these comparisons, field by field, with the mean results from series A and C has already been

TABLE VIII

DETAILED RESULTS FOR INDIVIDUAL STARS

Field	Curve	Pl. Corr.	Z. Pt. Adj.	Mag.	v	Field	Curve	Pl. Corr.	Z. Pt. Adj.	Mag.	v
Kopff 522						Kopff 659					
9	8.75	+34	+9	9.18	+ 9	13	9.20	+31	−6	9.45	+ 1
10	9.33	−12	+6	9.27	0	12,13	8.90	+43	−4	9.29	+18
9,10	9.19	+17	+9	9.45	−15		9.00	+34	−6	9.28	
	9.16	+16	+6	9.38		14	9.80	−22	−3	9.55	− 9
10,11	9.52	−42	+6	9.16	+ 5	13,14	9.59	+ 2	−6	9.55	−10
	9.59	−34	+2	9.27			9.54	+ 5	−3	9.56	
Mean				9.27						9.46	
Red. for color				−0.01						+ 0.12	
MW pg. mag				9.26						9.58	
Yerkes mag				8.79						10.02	
MW − *Y*				+0.47						− 0.44	
Kopff 531						Kopff 667					
10	8.23	+ 8	+6	8.37	−27	14	8.86	+30	− 3	9.13	+ 9
11	8.36	−30	+2	8.08	+ 2	15	9.16	− 5	+21	9.32	−10
10,11	8.00	−13	+6	7.93	+10	14,15	9.08	+17	− 3	9.22	0
	8.17	−13	+2	8.06			8.82	+18	+21	9.21	
(10,11)s	11.09	− 9	+6	8.05	+15						
	10.92	− 9	+2	7.84							
Mean				8.10						9.22	
Red. for color				+0.03						+ 0.13	
MW pg. mag				8.13						9.35	
Yerkes mag				7.64						9.83	
MW − *Y*				+0.49						− 0.48	

given in the last two columns of Table III. For the present test, however, it is necessary to know the relation of the Yerkes magnitudes to the results obtained from individual plates of series A.

The mean values of $A - Y$ for each A plate are given in the second and sixth columns of Table IX. These means reproduce the systematic progression shown by the results in Table VI in a form which admits a comparison with similar differences referred to series B. Since the zero points for series C are the same as for A, the differences in the last two columns of Table III have the form $A - B$, and as

such are transferred to the fourth and eighth columns of Table IX. Combined with the corresponding values of $A-Y$, they give the desired values of $B-Y$. These show that when referred to the zero points of series B, the Mount Wilson magnitudes, especially the photographic series, diverge from the Yerkes measures in substantially the same way as when referred to series A. The mean discordance for the photographic magnitudes, which is the more serious of the two, is

$A-Y$	$B-Y$	
+0.22	+0.25	(Fields 5, 6, 7)
−0.16	−0.21	(Fields 1, 2, 3)

TABLE IX

AGREEMENT OF YERKES MAGNITUDES WITH INDIVIDUAL POLAR COMPARISONS OF SERIES A AND SERIES B

(Unit=0.01 mag.)

FIELD	PG.				PV.			
	$A-Y$	No. Stars	$A-B$	$B-Y$	$A-Y$	No. Stars	$A-B$	$B-Y$
7	+25	5	− 2	+27	−4	1	+ 7	−11
6	+31	23	−12	+43	+5	23	− 9	+14
5	+14	33	+ 2	+12	+4	32	−17	+21
4	− 6	28	−13	+ 7	+7	27	− 5	+12
3	−17	30	0	−17	+2	29	+ 4	− 2.
2	−18	28	+15	−33	−6	28	+12	−18
1	−13	28	− 1	−12	−7	28	+ 3	−10

The general agreement of the A and B series does not necessarily mean that the large relative error for the two groups of fields is to be attributed wholly to the Yerkes magnitudes. Although the photographs for the two series of Mount Wilson comparisons were usually made on different nights, peculiar sky conditions may have introduced similar errors into each. Among other causes sky illumination from lights in the valley to the southwest of Mount Wilson is a possible source of systematic error; but in the present case the fields were all observed at so nearly the same altitude as to make a relative error of 0.4 mag. wholly inadmissible.

Color indices in different fields for stars of same spectral type.—An indication as to the location of the error may be found by comparing the color indices for the two groups of fields with the spectral types

of the stars. The differences in the Mount Wilson and Yerkes color indices have the form

$$C - C_1 = (Pg - Pg_1) - (Pv - Pv_1) \qquad \text{(Fields 5, 6, 7)},$$
$$C' - C_1' = (Pg' - Pg_1') - (Pv' - Pv_1') \qquad \text{(Fields 1, 2, 3)}.$$

The values of the first of the parentheses in these formulae are +0.22 and −0.16, respectively. The corresponding photovisual differences from the sixth column of Table IX are +0.05 and −0.04. Hence

$$(C - C') - (C_1 - C_1') = \Delta C = +0.29 \text{ mag.}$$

The two differences in the left-hand member of this relation, which must be satisfied by the color indices of stars of the same spectral type, indicate the apportionment of error between the Mount Wilson and Yerkes magnitudes.

The results are collected in Table X, which gives for the two groups of fields and the two series of observations the mean color indices corresponding to the spectral types in the first column. The values of ΔC in the last column agree well and show that the effect of accidental errors is unimportant. The mean ΔC is less than the 0.29 found from the individual polar comparisons because it depends on color indices derived from the final magnitudes. Since these include the zero-point determination for the transition field No. 4, the discordance is partly smoothed out (see Table IX).

The values of ΔC check the consistency of the results, but the feature of present interest is their origin. The sequences of differences in the sixth and twelfth columns, and more particularly their mean values, +0.07 and −0.17 mag., indicate that 0.3 of the discordance originates in the Mount Wilson measures and 0.7 in the Yerkes results.

This conclusion depends on a comparison of observed colors for different groups of stars which supposedly have the same color. Errors of classification, lack of discrimination between giants and dwarfs, and dispersion in color for the same type all tend to invalidate the supposition. The single M-type star in the second group of fields, for example, if retained, would seriously modify the distribution of error. Its color index is 1.0 mag.; but no M star, either giant

or dwarf, as far as we know, has so small a color index. The spectral type doubtless is in error. For the other spectral groups the number of stars is generally sufficient to eliminate most of the accidental errors of classification. Systematic differences between the two groups of fields may, however, affect the result. Comparison of the frequencies of spectral types for these groups with those for several other groups of stars selected from Tables IV and V shows a general agreement, with a maximum at Ko in all cases except for fields 5, 6, 7, for which it occurs at G7. The frequency-curve for this group would agree better with the others if 0.3 of an interval could be added to the spectral types for most of the stars. If this difference in frequencies is real, the results in Table X require no change;

TABLE X

COMPARISON OF COLORS FOR STARS OF SAME SPECTRAL TYPE

HD Sp.	Mount Wilson						Yerkes						ΔC
	C	No.	C'	No.	$C-C'$	Wt.	C_1	No.	C_1'	No.	C_1-C_1'	Wt.	
A2	+0.08	5	−0.12	2	+0.20	1	−0.19	4	−0.10	2	−0.09	1	+0.29
F4	0.30	10	+ .16	14	+ .14	6	+0.14	10	+0.26	14	− .12	6	.26
G3	0.50	7	.53	8	− .03	4	0.34	5	0.60	8	− .26	3	.23
K2	1.01	10	.96	23	+ .05	7	0.89	8	1.08	23	− .19	6	.24
M	+1.67	6	+0.91	1	(+0.76)	0	+1.58	6	1.07	1	(+0.51)	0	+0.25
Weighted mean	.....	....	.....	....	+0.07	....	.....	....	.....	....	−0.17	....	+0.25

but if it represents an error in classification, the values of C and C_1 should be decreased by about 0.1 mag. This would improve the agreement for the Mount Wilson color indices, but make that for the Yerkes values less satisfactory.

9. RELATION BETWEEN COLOR INDEX AND SPECTRAL TYPE

Table XI gives the mean color index for each spectral type based on all the stars in Tables IV and V for which spectra are available.

The large positive color index for the single B2 star (H.D. 83953, Table V) seems to be real; but in view of the discordances shown in Table VIII confirmation is required. The star is I Hydrae. The *HD* remarks: "The line $H\beta$ is a narrow, bright line superposed on a hazy, dark band. Other lines are dark." Merrill classifies it as B2e and finds it to be a normal object of its kind. A short-focus camera plate of the region by Hubble gives no indication of surrounding nebulosity.

The general relation between color and spectrum expressed by Table XI is essentially the same as that found by Ross and Zug. The Mount Wilson color indices are systematically a few hundredths smaller than the Yerkes values, as might be expected from the relation between zero points shown in Table VII. The most noteworthy features of these results, however, are, first, that both series of measures show a color index of about −0.15 mag. for Ao stars, whereas the value for the polar region is zero; and, second, that both depart widely from the accordant relations between color and spectrum found by King, Parkhurst, and Seares. Some of this last difference is doubtless to be accounted for by the presence of dwarfs among the fainter comparison stars for Eros, but this cannot wholly explain the divergence. These questions will be discussed in a later *Contribution*.

TABLE XI

MEAN COLOR INDEX AND SPECTRUM

Sp.	*C*	No. Stars	Sp.	*C*	No. Stars	Sp.	*C*	No. Stars
B2p	+0.51	1	A5	−0.02	9	G5	+0.63	33
B5	− .34	1	A7	− .18	1	K0	+0.89	66
B8	− .26	2	F0	+ .10	26	K2	+1.11	29
B9	− .19	5	F2	+ .11	13	K5	+1.21	31
A0	− .14	21	F5	+ .29	27	K7	+1.61	2
A2	− .08	13	F8	+ .26	33	M	+1.53	16
A3	−0.03	3	G0	+0.37	27	Total	.....	359

10. COMPARISONS WITH HARVARD MAGNITUDES

Intercomparison of the various Harvard catalogues has shown that they differ appreciably in scale and in color equation.[1] The general catalogues, *HA* **50, 54,** and especially the *Draper Catalogue*, which is a compilation from other volumes of the *Harvard Annals*, are therefore not homogeneous, and at present it is impossible to say what correction should be applied to any given star to reduce it to the international system. To utilize fully this rich observational

[1] See, for example, *Harvard Annals*, **64**; Müller and Kempf, *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, **17**, 1907; Zinner, *Veröffentlichungen der Remeis-Sternwarte zu Bamberg*, **2**, 65, 1926; Seares, *Mt. Wilson Contr.*, No. 288; *Astrophysical Journal*, **61**, 284, 1925.

material it will be necessary to discuss the original magnitudes, reduce them to the international system, and then recombine them into mean catalogue values. The photovisual magnitudes in Tables IV and V permit a few comparisons of this sort—not enough for final conclusions, but sufficient to show the importance of the corrections.

The differences $MW-H$ thus found for different Harvard sources were grouped according to Harvard magnitude and plotted against MW color indices. Graphical solutions then gave the linear expressions collected in Tables XII and XIII. These relate to measures made by Pickering, Wendell, and Bailey with the 2-inch, 4-inch, and 12-inch meridian photometers. A few observations by Searle were also found in *HA* **14**, but they indicate little more than the presence of a considerable color equation. The absence of a color coefficient does not mean that the coefficient is zero, but merely that the data do not permit a satisfactory determination.

In any one of the groups of relations in Tables XII and XIII the sequence of first terms represents the scale divergence after the Harvard measures have been freed from color in accordance with the coefficients of the second terms. For the first two groups the color coefficients of Pickering and Wendell are approximately the same, and mean values are used in Table XI; but this cannot be done for the first terms. For these Pickering's values are given, with values of $P-W$ following in parentheses. For results in *HA* **14**, the mean zero-point difference $P-W$ is -0.17; for *HA* **24**, $+0.14$.

The significant features of the comparisons are: (1) the progression in the scale difference shown by all the measures of Pickering and Wendell and the size of the color coefficient affecting their results; (2) the increase in the color coefficient with magnitude and the systematic differences between the coefficients for the three photometers; (3) the difference in the scale corrections for Bailey and Pickering and the relative freedom of Bailey's measures from the influence of color.

It is not suggested that the results in Tables XII and XIII are final or that they are applicable to all portions of the catalogues involved. The data on which they are based are far too slender for that. The characteristic features are revealed so clearly, however,

TABLE XII

COMPARISONS WITH DIFFERENT HARVARD CATALOGUES ($MW-H$)

HARV. VIS. MAG.	2-INCH M.P.		4-INCH M.P.						12-INCH M.P.	
	HA 14, *P* and *W*	No. Obs.	*HA* 24, 1, 4, *P* and *W*	No. Obs.	*HA* 44, *P*	No. Obs.	*HA* 45, *P*	No. Stars*	*HA* 70, 74, *P*	No. Stars*
3.9	−0.16(+0.13)	6	.....	....	−0.13	3	−0.12	1	.....	.....
4–5	+ .10(− .17)	16	−0.30(+0.15)	9	− .25	7	− .08−0.10C	3	.....	.....
5–6	+ .20(− .20)−0.32C	33	+ .04(+ .16)−0.15C	26	− .04−0.23C	33	+ .07− .20C	5	.....	.....
6–7	+0.30(−0.25)−0.46C	28	+ .10(+ .14)− .38C	37	+0.07−0.35C	39	+ .11− .27C	33	.....	.....
7–8	.....	....	+ .15 −0.40C	7	.....	....	+0.25−0.33C	26	−0.10	1
8–9	.....	....	+0.25(+0.13)	16	.....	....	.....	.....	+0.20−0.30C	16

TABLE XIII

COMPARISON WITH BAILEY ($MW-B$)

HARV. VIS. MAG.	4-INCH M.P.			
	HA 44, *B*	No. Obs.	*HA* 46, *B*	No. Stars*
3.8	−0.06	3	−0.05	1
4–5	− .18	1	− .19	2
5–6	− .10−0.10C	10	− .12+0.00C	9
6–7	0.00−0.10C	17	−0.16	3

* The number of observations is roughly three times the number of stars.

as to indicate that reliable formulae for reduction to a homogeneous system may be obtained with comparatively little effort.

The diversity in the corrections shown in Tables XII and XIII lessens the importance of any comparison with the mean Harvard magnitudes given in the *Draper Catalogue*. The general agreement with a similar comparison made some years ago for the stars of the Polar Sequence[1] is, nevertheless, of interest.

The results, after a slight smoothing, are as given in the third column of Table XIV. The comparison with the polar sequence is in the following column. The divergence in the color coefficients probably arises from differences in instruments and observers concerned in the two groups of Harvard magnitudes. The scale difference, which is mainly in the zero point and amounts to about 0.05 mag., may also arise partly from this source.

TABLE XIV

COMPARISON WITH MEAN HARVARD VISUAL SCALE

Harv. Vis. Mag.	No. Stars	*MW−H* Present Results	*MW−H* Polar Sequence	Differences
3.8.....	1	−0.05−(0.10)C	−0.08−0.09C	−0.03+0.01C
4–5.....	3	− .10−(.13)C	− .07− .10C	+ .03+ .03C
5–6.....	13	− .01− .17 C	− .04− .12C	− .03+ .05C
6–7.....	42	+ .08− .21 C	+ .02− .13C	− .06+ .08C
7–8.....	28	+ .17− .26 C	+ .10− .14C	− .07+ .12C
8–9.....	12	+0.25− 0.36 C	+0.19−0.15C	−0.06+0.21C

Magnitudes given in the *Draper Catalogue* to a single decimal are from the *Durchmusterung* catalogues, reduced to the *HD* system by the application of mean corrections. The catalogues involved are the *SDM*, the *CD*, and the *CPD*, the last only for stars south of −16° declination. Comparisons with these catalogues are given in Table XV. *HD* magnitudes printed in italics have not been used.

For none of the three catalogues is there any certain evidence of a progression in the values of *MW−H*. As far as the present data go, the only difference is in the zero points, for which the means are +0.01, +0.11, and +0.12 mag., respectively. The parallelism of scales is in surprising contrast to the 34 per cent divergence found

[1] Seares, *Mt. Wilson Contr.*, No. 288, Table II; *Astrophysical Journal*, **41**, 284, 1925

by Ross and Zug[1] in a similar comparison of visual magnitudes. Nearly all their stars, however, are from the northern *Durchmusterung* (with Harvard corrections); and the discrepancy between their results and ours apparently represents a real difference in the magnitude systems for the northern and southern catalogues.

TABLE XV

COMPARISON WITH *DM* MAGNITUDES REDUCED TO *HD* SYSTEM

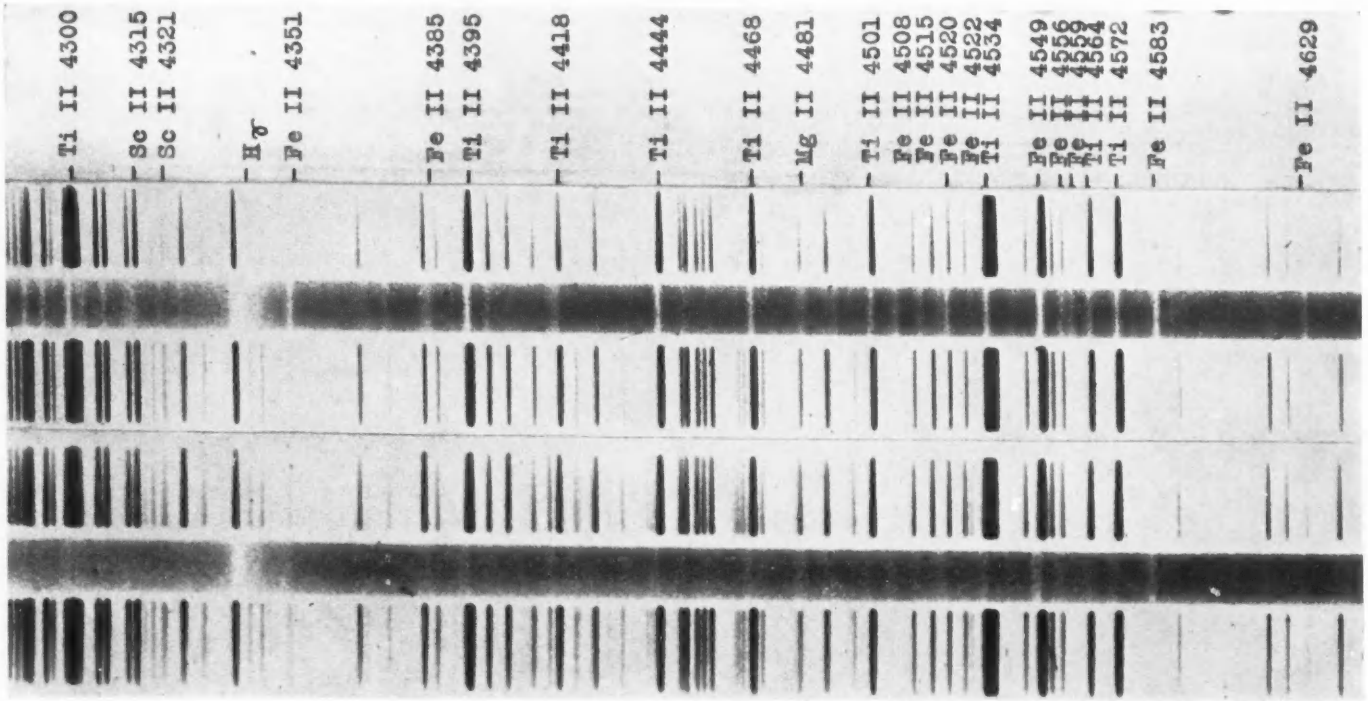
HARV. MAG.	*MW−H(SDM)*			*MW−H(CD)*			*MW−H(CPD)*		
	S.D.	A.D.	No.	S.D.	A.D.	No.	S.D.	A.D.	No.
7.5– 7.9	−0.15	±0.22	6	.....	.....	.....	.....	.....	.....
8.0– 8.4	− .02	.35	13	+0.22	±0.40	9	+0.20	±0.20	7
8.5– 8.9	− .05	.26	45	+ .04	.32	17	+ .05	.18	20
9.0– 9.4	+ .11	.28	49	+ .15	.27	17	+ .14	.23	37
9.5– 9.9	−0.07	±0.12	9	+0.02	±0.10	5	+ .18	.29	32
10.0–10.4	.....	.....	.....	.....	.....	.....	−0.03	±0.17	14
Means and sums	+0.01	±0.27	122	+0.11	±0.30	48	+0.12	±0.23	110

The approximate character of the *Durchmusterung* magnitudes means a large scatter in the values of *MW−H* for individual stars, and it is therefore difficult to speak definitely of the influence of color. The *CD* and *CPD* show nothing; the representation of the *SDM* differences would be somewhat improved by adopting a color coefficient of 0.2 to 0.4, but the correlation is uncertain and the results of Table XV are preferred for the present.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
November 1930

[1] *Loc. Cit.*

PLATE XIV



SPECTRUM OF 17 LEPORIS Jan. 2, 1929 (top) and Oct. 20, 1930 (bottom); enlarged 14.3 times. Titanium and iron sparks were used for the comparison spectrum.

RECENT CHANGES IN THE ABSORPTION SPECTRUM OF 17 LEPORIS

By OTTO STRUVE

ABSTRACT

The spectroscopic binary *17 Leporis is shown to undergo profound changes in the character of its absorption spectrum.* In 1925 and again in 1928–1929 the spectrum contained many strong lines of *Ti* II, *Fe* II, *Sc* II, *H*, and weaker lines of *Mg* II, *Fe* I, and of other elements. Recent plates taken in October, 1930, show no lines of *Fe* I. The lines of *Sc* II and of *Ti* II are greatly weakened, while those of *Fe* II are weakened in a smaller degree. The lines of *Mg* II and of *H* have not changed appreciably. Many of the stronger lines appear as close doubles on the recent plates. The hydrogen lines are peculiar. The radial velocity derived from the *Mg* II line differs from that derived from the other lines.

In the course of our work on radial velocities of A-type stars, I noticed, several years ago, that the star 17 Leporis appeared to have a variable spectrum: on a plate taken on December 6, 1912 very few absorption lines could be seen, while two other plates, of January 23 and of February 1, 1925, displayed a spectrum with many unusually strong lines of *Ti* II and *Fe* II. A note to this effect was published in the Yerkes catalogue of radial velocities of A-type stars.[1] Unfortunately the first plate was badly underexposed and could not be used for a more detailed study of the spectral variations.

Additional spectrograms were secured on December 24 and 28, 1928, and on January 2, 16, and 30, 1929, but all of them resembled the spectra taken in 1925. However, on a plate taken by Mr. P. C. Keenan on October 19, 1930, I find that the star has reverted to the type of spectrum having weak lines. To confirm this two additional plates were taken, one on October 20 by Mr. W. W. Morgan and the other on October 22 by myself. This remarkable change in the character of the spectrum between the years 1929 and 1930 is shown in the reproductions herewith (Plate XIV). The original spectrograms were taken with the Bruce spectrograph attached to the 40-inch refractor, and have a linear dispersion of 30 A per millimeter at λ 4500. The two spectra were taken on Eastman-40 plates and were both developed in Eastman D 11. They were treated in exactly

[1] *Publications of the Yerkes Observatory*, **7**, Part I, 28, 1929.

the same manner in making the enlargements. The 1930 plate shows only faint lines of *Ti* II and *Fe* II, while the 1929 plate resembles the spectrum of the supergiant 7 ϵ Aurigae, although the spectral type of 17 Leporis is distinctly the earlier of the two, about A8. The change is readily seen by comparing the ratio of the intensities of two lines, such as *Ti* II λ 4468 and *Mg* II λ 4481, on one plate with the ratio of the intensities of the same lines on the other plate: In 1929 the titanium line is very much stronger than the magnesium line, while in 1930 magnesium is slightly stronger than titanium. A preliminary study of the plates discloses the following facts:

1. Various elements are affected in various degrees by the spectral changes. The greatest change in intensity is recorded for the *Sc* II lines: Thus λ 4247 (not shown in the plate) is very strong in 1929 and very weak in 1930. The enhanced titanium lines, *Ti* II, are affected nearly as much (e.g., $\lambda\lambda$ 4468, 4501, 4564, 4572, etc., in the plate). Several arc lines of *Fe* I (λ 4045, etc.) are well visible in 1929, but cannot be discerned in 1930. The enhanced iron lines, *Fe* II, are affected to a less extent than the *Ti* II lines. This is shown by comparing *Ti* II λ 4501 and *Fe* II λ 4508: In 1929 the *Ti* II line is much the stronger, while in 1930 the intensities are about the same. The line *Mg* II λ 4481 is of the same intensity on both plates, and this is also true of the hydrogen lines.

2. Several strong lines of *Ti* II and of *Fe* II are close doubles in 1930, while they are single on the plate of 1929. The violet component is slightly stronger than the red. The *Sc* II line at λ 4247 is also double, but in this case the red component is much stronger. It is reasonably certain that the splitting into two components does not account for the general weakness of the lines in 1930. Thus *Mg* II λ 4481, which also shows traces of doubling or broadening, is equally strong on both plates. Furthermore, the sum of the intensities of the two narrow components in 1930 is not sufficient to equal the great intensity of the lines in 1929.

3. The radial velocity from the *Mg* II line λ 4481 does not agree with that obtained from the lines of *Ti* II and *Fe* II. This may be seen from the reproduction, since the wave-length of the titanium comparison line (λ 4481.251) coincides almost exactly with that of the *Mg* II line (λ 4481.230). In 1929 the displacement is a little to-

ward the red. At the same time the *Ti* II lines (λλ 4444, 4468, 4501, etc.) give large negative displacements. A similar difference is observed in 1930.

4. The hydrogen lines are peculiar: a very strong core of absorption, resembling the deep hydrogen lines in ε Aurigae, is unsymmetrically superimposed over a broad and hazy absorption band.

It may be noted that 17 Leporis was announced as a spectroscopic binary by R. E. Wilson at the Lick Observatory.[1] The range in velocity from the Lick and Yerkes spectrograms is more than 60 km/sec., but no orbit has been derived.[2] Wilson had also noted the discrepancy in radial velocity between the lines of *Mg* II and other elements. Our plates do not, however, confirm the existence of oxygen lines in this star. J. Stebbins used 17 Leporis as a comparison star in his photo-electric observations of β Canis Majoris, but no variation was found over a period of 34 days.[3] No other observations of the brightness of 17 Leporis are known to me.[4] The position of 17 Leporis for 1900 is $\alpha = 6^h0^m.5$, $\delta = -16°29'$; the spectrum was classified at Harvard as A0, and the photometric magnitude was given at 5.04. A more detailed discussion of the observations, including the radial velocities, will be reserved until more material will be available.[5]

YERKES OBSERVATORY
October 22, 1930

[1] *Lick Observatory Bulletins*, **8**, 81, 1914.

[2] *Publications of the Lick Observatory*, **16**, 85, 1928; *Publications of the Yerkes Observatory*, **7**, Part I, 28, 1929.

[3] *Publications of the Washburn Observatory*, **15**, 75, 1928.

[4] M. Güssow und P. Guthnick, *Kleinere Veröffentlichungen der Universitäts-Sternwarte zu Berlin-Babelsberg*, No. 8, 1930.

[5] Since this note was written, seven more spectrograms of 17 Leporis have been obtained, mostly with the dispersion of three prisms. The intensity of the enhanced lines has gradually increased, and in December 1930 the star was definitely approaching the type of spectrum illustrated in the plate, for January 2, 1929. There is a suspicion of an emission line on the red side of the absorption line *Hβ*. The three-prism spectrograms show interesting changes in the contours of the absorption lines.

REVIEWS

Theory of the Potential. By W. D. MacMillan. McGraw-Hill Book Co. Pp. xiii+469. Figs. 111. $5.00.

In 1886 B. O. Pierce published a small volume on the Newtonian potential function. In the forty-one years following that date, no book on potential theory was published in this country or, for that matter, in the English language. It should be a matter of pride that we now have three splendid books on potential theory by American authors. These three are *The Logarithmic Potential, Discontinuous Dirichlet and Neuman Problems*, by G. C. Evans (1927); *Foundations of the Potential Theory*, by O. D. Kellogg (1929); and the volume here under review, *Theory of the Potential*, by W. D. MacMillan. In interest of content, in sound scholarship, and in clear exposition all three of these books are real additions to that growing literature which proves that American science has come of age. Incidentally, the three volumes supplement one another almost as accurately and completely as though the three authors had agreed in advance on the boundaries of their own preserves.

Professor MacMillan's *Theory of the Potential* forms the second volume of his treatise on theoretical mechanics. The spirit and purpose of the first volume—*Statics and the Dynamics of a Particle*—carry over successfully to this second volume. The author has, by his own statement, a dual purpose, "to give the reader a connected account of a certain field of mechanics which is very useful from the point of view of a physicist and very beautiful from the point of view of a mathematician." Professor MacMillan has been successful in fulfilling his purpose. He has the peculiar ability, perhaps derived from his familiarity with the older natural philosophers, to deal with such a subject in an intensely practical way without robbing it of any of the beauty of its analytical structure. The book has more really interesting practical examples and illustrations than any other volume on this subject familiar to the reviewer. On every page the reader feels himself rubbing shoulders with nature, and yet this practical aspect is not gained, as some authors falsely believe can be done, by using loose arguments or by dodging analytical difficulties. The author is too able to have to content himself with a middle ground, and there is probably no other book on the subject which is, so to speak, both as

physical and as mathematical as is this one. It contains, for example, a considerable amount of highly interesting mathematical material which is not found in Poincaré's well-known treatise, and it contains many concrete cases and important illustrations also not found in the French classic.

In the first two chapters the reader studies the attraction and potential due to a large number of special line, surface, and volume distributions of matter, such as a straight line, a circular arc, a circular disk, a rectangular plate, an ellipsoidal homeoid, the frustum of a cone, a sphere or spherical shell, a solid ellipsoid, a spheroid, an elliptic cylinder, a right parallelopiped, etc. Along with these special examples one becomes acquainted with the simpler of the fundamental definitions and theorems relating to the Newtonian potential function, the existence of it and of its partial derivatives, its behavior at infinity, etc. One notes here, as throughout the book, the fine balance of theory and practice, of old and new. In the study of ellipsoids, the classical results of Ivory and MacLaurin are given; and at the same time the complicated expressions for the potentials are all reduced to forms practical for actual computation before the discussion is concluded.

The third chapter gives an excellent development of the analytical specification of vector fields, including the theorems of Green and of Gauss and their applications to the study of harmonic functions. Chapters iv and v are chiefly concerned with surface distributions of matter. The study of the potential due to such distributions and of the first and second derivatives follows, in general, the lines laid down by Poincaré. The author devotes, however, considerable space to the study of level layers and their properties, since he later intends to base his general existence proofs on the existence theorem for a level layer on a given surface. Chapter vi studies double-layer distributions, partly on account of the importance of this theory in electrical problems, and also because of the application of the ideas to the proof of Dirichlet's principle. This fundamental principle is discussed at length, the proofs of Neumann and Poincaré being presented in detail and with great clarity. One may remark, relative to this central problem of potential theory, that Professor Kellogg's book treats this whole question from the point of view of integral equations; while Professor Evans, who is almost exclusively concerned with this same topic, is interested (as is also Professor Kellogg) in gaining as great as possible generality for this principle by weakening as far as possible the hypotheses under which it holds. Chapter vii contains an exposition of the theory of spherical harmonics, while in chapter viii is brought together

what the reviewer believes will prove to be an exceedingly useful treatment of ellipsoidal harmonics.

The book is well printed. There are over one hundred well-drawn figures. Only a very few and entirely unimportant typographical errors were noted. It appears to be a very teachable volume, although few classes will be able to accept all it offers. There are problems for the student to work, but only fifty-five of them. This trivial criticism is the only adverse one the reviewer has to make, for this is a complete, a scholarly, and a satisfying book.

WARREN WEAVER

UNIVERSITY OF WISCONSIN

CORRIGENDUM

In Plate XI of this volume, facing page 187, in the article by C. D. Higgs entitled "Note on the Variable Lines of Hydrogen in the Spectrum of 52 π Aquarii," the first stellar spectrum represents the normal Bo star 47 ρ Leonis. The dates for the spectrograms of 52 π Aquarii are, reading from the top, for $H\beta$: 1915 July 30; 1919 Nov. 17; 1928 Sept 16; 1928 Sept. 20; 1930 May 16; 1930 Aug. 21; 1930 Sept. 6. For $H\gamma$ they are 1915 July 30; 1919 Nov. 17; 1928 Sept. 16; 1928 Sept. 20; 1930 Aug. 21; 1930 Sept. 6. The spark spectra of *Ti* and *Fe* were used for comparison. All the spectra are reproduced as negatives.

Also in Figure 1, page 188, abscissae are in Ångstroms and ordinates are percentages of emission or absorption.

INDEX TO VOLUME LXXII

SUBJECTS

PAGE

Absolute Magnitudes from the Distributions of Parallactic Motions and Radial Velocities, A Method for Determining the Distribution of Stellar. *Gustaf Strömberg* 111

52 π Aquarii, Note on the Variable Lines of Hydrogen in the Spectrum of. *C. D. Higgs* 187

U Aquilae, DT Cygni, and W. Geminorum, Variations in Radial Velocity of the Cepheid Variables. *Roscoe F. Sanford* 46

Bright-Line Stars of Classes B and A, Notes on the Spectra of Ten. *Paul W. Merrill* 98

α^2 Canum Venaticorum, On the Variation of the Intensities of Spectral Lines in. *A. Markov* 301

RS Canum Venaticorum, The Spectrographic Orbit of. *Alfred H. Joy* . 41

Carbon Isotope of Mass 13, Evidence from Band Spectra of the Existence of. *Arthur S. King* and *Raymond T. Birge* 20

U Cephei: An Anomalous Spectrographic Result. *Edwin F. Carpenter* . 205

Cepheid Variable W Geminorum, U Aquilae, and DT Cygni, Variations in Radial Velocity of the. *Roscoe F. Sanford* 46

Changes in the Absorption Spectrum of 17 Leporis, Recent. *Otto Struve* . 343

Chromosphere, Heights in the. *S. A. Mitchell* 146

Classification of the Spectra of Europium, Gadolinium, Terbium, Dysprosium, and Holmium, λ 3850 to λ 4700. *Arthur S. King* 221

Contours of the Hydrogen Lines in A Stars, Systematic Errors in the Determination of the. *Emma T. R. Williams* 127

Corrigendum. *C. D. Higgs* 348

DT Cygni, Variations in Radial Velocity of the Cepheid Variables W Geminorum, U Aquilae. *Roscoe F. Sanford* 46

Dysprosium, and Holmium, λ 3850 to λ 4700, Temperature Classification of the Spectra of Europium, Gadolinium, Terbium. *Arthur S. King* . 221

Eros, Magnitudes of Southern Comparison Stars for. *F. H. Seares, B. W. Sitterly,* and *Mary C. Joyner* 311

Errors in the Determination of the Contours of the Hydrogen Lines in A Stars, Systematic. *Emma T. R. Williams* 127

Europium, Gadolinium, Terbium, Dysprosium, and Holmium, λ 3850 to λ 4700, Temperature Classification of the Spectra of. *Arthur S. King* . 221

Gadolinium, Terbium, Dysprosium, and Holmium, λ 3850 to λ 4700, Temperature Classification of the Spectra of Europium. *Arthur S. King* 221

PAGE

Gamma Geminorum (Class Ao) Spectrum of. *Sebastian Albrecht* . . 65
W Geminorum, U Aquilae, and DT Cygni, Variations in Radial Velocity of the Cepheid Variables. *Roscoe F. Sanford* 46
Helium, New Ultra-Violet Spectrum of. *John J. Hopfield* 133
Holmium, λ 3850 to λ 4700, Temperature Classification of the Spectra of Europium, Gadolinium, Terbium, Dysprosium, and. *Arthur S. King* . 221
Hydrogen in the Spectrum of 52 S Aquarii, Note on the Variable Lines of. *C. D. Higgs* 187
Hydrogen Lines in A Stars, Systematic Errors in the Determination of the Contours of. *Emma T. R. Williams* 127
Hydrogen Lines and Their Relation to the Stark Effect, A Study of Stellar. *C. T. Elvey* and *Otto Struve* 277
Intensities in the Solar Spectrum, Multiplet. *R. van der R. Woolley* . 256
Intensities of Spectral Lines in α^2 Canum Venaticorum, On the Variation of the. *A. Markov* 301
Interstellar Calcium Lines in the Eclipsing Binary U Ophiuchi, The Coexistence of Stellar and. *Otto Struve* 199
Isotope of Mass 13, Evidence from Band Spectra of the Existence of a Carbon. *Arthur S. King* and *Raymond T. Birge* 20
17 Leporis, Recent Changes in the Absorption Spectrum of. *Otto Struve* . 343
M Stars Brighter than the Sixth Apparent Magnitude as Determined from Parallactic and Peculiar Velocities, The Distribution of Absolute Magnitudes among. *Gustaf Strömberg* 117
Magnitudes from the Distributions of Parallactic Motions and Radial Velocities, A Method for Determining the Distribution of Stellar Absolute. *Gustaf Strömberg* 111
Method for Determining the Distribution of Stellar Absolute Magnitudes from the Distributions of Parallactic Motions and Radial Velocities. *Gustaf Strömberg* 111
Objectives for Astronomical Spectrographs, Two High-Speed Camera. *W. B. Rayton* 59
Orbit of RS Canum Venaticorum, Spectrographic. *Alfred H. Joy* . . 41
Reviews:
Baker, Robert H. *Astronomy, an Introduction* (W. F. Long) . . 131
Bloch, Eugène. *L'ancienne et la nouvelle théorie des quanta.* (O. Struve) 64
Faraday Society. *Molecular Spectra and Molecular Structure* (Andrew Christy) 63
Goudsmit, Samuel, and Linus Pauling. *The Structure of Line Spectra* (Carl Eckart) 62
Larmor, Sir Joseph. *Mathematical and Physical Papers* (F. E. Carr) . 202
MacMillan, W. D. *Theory of the Potential* (Warren Weaver) . . 346
Pauling, Linus, and Samuel Goudsmit. *The Structure of Line Spectra* (Carl Eckart) 62

PAGE

Rotation of Stars, On the Axial. *Otto Struve* 1
Si III, Intensities in Stellar Spectra of a Triplet of. *Otto Struve* and *C. T. Elvey* 267
Spectra, Absorption Lines of Sulphur (*S* III and *S* II) in Stellar. *F. E. Roach* 191
Spectra of Europium, Gadolinium, Terbium, Dysprosium, and Holmium, λ 3850 to λ 4700, Temperature Classification of the. *Arthur S. King* . 221
Spectra of the Existence of a Carbon Isotope of Mass 13, Evidence from Band. *Arthur S. King* and *Raymond T. Birge* 20
Spectra of Ten Bright-Line Stars of Classes B and A, Notes on. *Paul W. Merrill* 98
Spectra of a Triplet of *Si* III, Intensities in Stellar. *Otto Struve* and *C. T. Elvey* 267
Spectrographic Orbit of RS Canum Venaticorum. *Alfred H. Joy* . . 41
Spectrographic Result, U Cephei: An Anomalous. *Edwin F. Carpenter* . 205
Spectrographs, Two High-Speed Camera Objectives for Astronomical. *W. B. Rayton* 59
Spectrum, Multiplet Intensities in the Solar. *R. van der R. Woolley* . . 256
Spectrum of 52 π Aquarii, Note on the Variable Lines of Hydrogen in. *C. D. Higgs* 187
Spectrum of Gamma Geminorum (Class Ao). *Sebastian Albrecht* . . 65
Spectrum of Helium, New Ultra-Violet. *John J. Hopfield* 133
Spectrum of 17 Leporis, Recent Changes in the Absorption. *Otto Struve* 343
Stark Effect, A Study of Stellar Hydrogen Lines and Their Relation to the. *C. T. Elvey* and *Otto Struve* 277
Stars, On the Axial Rotation of. *Otto Struve* 1
Stars for Eros, Magnitudes of Southern Comparison. *F. H. Seares, B. W. Sitterly*, and *Mary C. Joyner* 311
Stellar Hydrogen Lines and Their Relation to the Stark Effect, A Study of. *C. T. Elvey* and *Otto Struve* 277
Stellar and Interstellar Calcium Lines in the Eclipsing Binary U Ophiuchi, The Coexistence of. *Otto Struve* 199
Sulphur (*S* III and *S* II) in Stellar Spectra, Absorption Lines of. *F. E. Roach* 191
Systematic Errors in the Determination of the Contours of the Hydrogen Lines in A Stars. *Emma T. R. Williams* 127
Terbium, Dysprosium, and Holmium, λ 3850 to λ 4700, Temperature Classification of the Spectra of Europium, Gadolinium. *Arthur S. King* 221
Ultra-Violet Spectrum of Helium, New. *John J. Hopfield* 133
Velocity of the Cepheid Variables W Geminorum, U Aquilae, and DT Cygni, Variations in Radial. *Roscoe F. Sanford* 46

INDEX TO VOLUME LXXII

AUTHORS

PAGE

ALBRECHT, SEBASTIAN. The Spectrum of Gamma Geminorum (Class Ao) 65

BIRGE, RAYMOND T., and ARTHUR S. KING. Evidence from Band Spectra of the Existence of a Carbon Isotope of Mass 13 20

CARPENTER, EDWIN F. U. Cephei: An Anomalous Spectrographic Result 205

CARR, F. E. Review of: *Mathematical and Physical Papers*, Sir Joseph Larmor 202

CHRISTY, ANDREW. Review of: *Molecular Spectra and Molecular Structure*, Faraday Society 63

ECKART, CARL. Review of: *The Structure of Line Spectra*, Linus Pauling and Samuel Goudsmit 62

ELVEY, C. T., and OTTO STRUVE. Intensities in Stellar Spectra of a Triplet of *Si* III 267

ELVEY, C. T., and OTTO STRUVE. A Study of Stellar Hydrogen Lines and Their Relation to the Stark Effect 277

HIGGS, C. D. Corrigendum 348

HIGGS, C. D. Note on the Variable Lines of Hydrogen in the Spectrum of 52 π Aquarii 187

HOPFIELD, JOHN J. New Ultra-Violet Spectrum of Helium 133

JOY, ALFRED H. The Spectrographic Orbit of RS Canum Venaticorum . 41

JOYNER, MARY C., F. H. SEARES, and B. W. SITTERLY. Magnitudes of Southern Comparison Stars for Eros 311

KING, ARTHUR S. Temperature Classification of the Spectra of Europium, Gadolinium, Terbium, Dysprosium, and Holmium, λ 3850 to λ 4700 . 221

KING, ARTHUR S., and RAYMOND T. BIRGE. Evidence from Band Spectra of the Existence of a Carbon Isotope of Mass 13 20

LONG, W. F. Review of: *Astronomy, an Introduction*, Robert H. Baker . 131

MARKOV, A. On the Variation of the Intensities of Spectral Lines in α^2 Canum Venaticorum 301

MERRILL, PAUL W. Notes on the Spectra of Ten Bright-Line Stars of Classes B and A 98

MITCHELL, S. A. Heights in the Chromosphere 146

RAYTON, W. B. Two High-Speed Camera Objectives for Astronomical Spectrographs 59

ROACH, F. E. Absorption Lines of Sulphur (*S* III and *S* II) in Stellar Spectra 191

PAGE

SANFORD, ROSCOE F. Variations in Radial Velocity of the Cepheid Variables W Geminorum, U Aquilae, and DT Cygni 46

SEARES, F. H., B. W. SITTERLY, and MARY C. JOYNER. Magnitudes of Southern Comparison Stars for Eros 311

SITTERLY, B. W., MARY C. JOYNER, and F. H. SEARES. Magnitudes of Southern Comparison Stars for Eros 311

STRÖMBERG, GUSTAF. The Distribution of Absolute Magnitudes among M Stars Brighter than the Sixth Apparent Magnitude as Determined from Parallactic and Peculiar Velocities 117

STRÖMBERG, GUSTAF. A Method for Determining the Distribution of Stellar Absolute Magnitudes from the Distributions of Parallactic Motions and Radial Velocities 111

STRUVE, OTTO. On the Axial Rotation of Stars 1

STRUVE, OTTO. The Coexistence of Stellar and Interstellar Calcium Lines in the Eclipsing Binary U Ophiuchi 199

STRUVE, OTTO. Recent Changes in the Absorption Spectrum of 17 Leporis 343

STRUVE, O. Review of: *L'ancienne et la nouvelle théories des quanta*, Eugène Bloch 64

STRUVE, OTTO, and C. T. ELVEY. Intensities in Stellar Spectra of a Triplet of *Si* III 267

STRUVE, OTTO, and C. T. ELVEY. A Study of Stellar Hydrogen Lines and Their Relation to the Stark Effect 277

WEAVER, WARREN. Review of: *Theory of the Potential*, W. D. MacMillan 346

WILLIAMS, EMMA T. R. Systematic Errors in the Determination of the Contours of the Hydrogen Lines in A Stars 127

WOOLLEY, R. VAN DER R. Multiplet Intensities in the Solar Spectrum . 256

Recent Astronomical

PHOTOGRAPHS
and
LANTERN SLIDES

Supplied from the

YERKES OBSERVATORY

Photographs taken by Professor Ross with his new doublet of high speed and wide angle:

N8. Orion and its nebulosities. Exposure time, 5 hours. See the *Astrophysical Journal* for April, 1927.

MW22. The Milky Way in Monoceros, showing bright and dark nebulae. Exposure, 8 hours.

N82. A region in Taurus showing the Pleiades, dark nebulae, and the nebula N.G.C. 1499.

N83. The exterior nebulosities of the Pleiades. Exposure, 10 hours.

Other interesting regions taken with this remarkable lens will soon be ready. Double slides are in preparation, showing a field taken with photovisual and photographic twin lenses. These indicate, by their contrast, the stars of high color.

Planetary photographs taken by Professor Ross with the 60-inch reflector at Mount Wilson:

P70. Venus, photographed in ultra-violet light, June 6, 24, and 26, 1927.

P71. Venus, 50 ultra-violet photographs on one slide, taken with the 60-inch and 100-inch reflectors, June 5 to July 27, 1927.

P72. Venus. An enlargement of the best exposure photographed on June 6, 1927, with ultra-violet light.

P37. Mars, made in the light of five different wave-lengths, from infra-red to ultra-violet. See the *Astrophysical Journal* for November, 1926.

P67. Mars. A comparison of the ultra-violet and the infra-red images of the foregoing.

P80. Jupiter, made in ultra-violet light, August 20, 1927.

Other photographs:

The Sun:

S18. The sun, showing unusual activity in both flocculi and prominences. A combination of calcium spectroheliograms of different dates (S6 and S90).

S19. The disk with many large spots, October 13, 1926.

S20. Enlargement of large spot of June 30, 1928.

S80. Enlargement of large spot group of September 21, 1926.

S81. Violent eruption of July 17, 1928, projected against disk.

S82. Enlarged view of violent eruption of July 17, 1928.

S84. Development of two brilliant eruptions as projected on the solar disk. July 17, 1928. Six exposures.

S85. Subsidence of second brilliant eruption on solar disk. July 17, 1928. Four exposures.

S86. Spectroheliogram of solar disk, May 2, 1928, showing well-marked double band of spots and flocculi.

S87. Dark calcium flocculus, July 17, 1928.

S150. Massive prominence ("The Bat"), June 7, 1928. Height 36,000 miles.

THE TWO SOLAR FAMILIES

The Sun's Children

By THOMAS C. CHAMBERLIN

In his last book Dr. Chamberlin has expounded more clearly and explicitly than ever before the features of the planetesimal hypothesis. Practically every peculiarity of movement which the earth and other planets present is given an explanation that follows logically from the primary principles of the hypothesis.

Professor Chamberlin takes as the important key to the interpretation of the genetic traits of the Planetary Family and the Cometary Family the recognition of their bi-parental genesis—one parent common to both and the other not. In this important contribution to our knowledge of the origin of the earth he traces the whole chain of parentages of our planet and its kin, as well as its wayward relatives, back to their stellar parentages.

$2.50, postpaid $2.60

THE UNIVERSITY OF CHICAGO PRESS

ANNUAL TABLES OF CONSTANTS AND NUMERICAL DATA

CHEMICAL, PHYSICAL, BIOLOGICAL AND TECHNOLOGICAL

Published under the supervision of the International Council of Research and the International Union of Pure and Applied Chemistry by the International Committee instituted by the Seventh Congress of Applied Chemistry (London, June 1909).

To the readers of the Astrophysical Journal:

The Volume VII of the Annual Tables is just published; it contains a chapter on "SPECTROSCOPY" particularly important. The division on emission spectra has been written by M. Bruninghaus and the one on absorption spectra by Professor Victor Henri; this latter part is an original work of considerable value.

This SPECTROSCOPY chapter consists of a special reprint of 514 pages the price of which is (bound copy) $7.60.

The readers of the Astrophysical Journal will receive a reduction of 50 per cent on the above price ($3.80) by sending their orders direct to the Secretary of the International Committee:

M. C. MARIE

9, rue de Bagneux, Paris VI^e^

accompanied by the amount in a check on a Paris bank; or a money order, in the name of M. C. Marie.

Important notices—*a*) In this chapter—as in the whole of Volume VII—all explanations to the tables are given in both English and French.

b) Reprints of Spectroscopy from previous volumes will also be sent to you with a 50 per cent reduction.